We will

- define *equivalence classes*,
- practice constructing them,
- notice how they relate to the full set, and
- prove a characterization of equivalence classes.
Definition (Equivalence Class)

The set \([x] = x/R = \{y | xRy\}\) is called the equivalence class of \(x\) in \(R\).

- \([x] = x/R\) is an alternate notation that derives from division (modulo arithmetic).
- Describe an equivalence class in the vernacular. You may wish to use the expression “alike.”
Consider \{ a, cactus, dessert, an, cringe, delight, apple, baboon, cart \}.

Two words are related if they begin with the same letter.

Find the equivalence class of “apple.”

Find the equivalence class of “cactus.”

Find equivalence classes for all the words.

How many distinct equivalence classes are there for this set and relation?

How many distinct equivalence classes are there for an English dictionary and this relation?
Remember $a \equiv b \mod n$ iff $n|(b - a)$.

- For this example it will help to remember the motivation for modulo arithmetic.
- Check if $4 \equiv 8 \mod 4$.
- Check if $3 \equiv 11 \mod 4$.
- Check if $2 \equiv 5 \mod 4$.
- Check if $7 \in [15]$.
- List the elements of $[1]$ for $\mod 4$. 
Find all integers equivalent to 2 mod 2.

Find all integers equivalent to 4 mod 2.

Prove that for all $y \in x/R$, $x/R = y/R$.

Find all integers equivalent to 1 mod 2.

Compare the equivalence classes of 1 and 2.

Prove that if $x \not\in x/R$, then $x/R \cap y/R = \emptyset$.

Compare your sets from above with $\mathbb{Z}$. 


Partition

A family $\mathcal{A}$ of subsets of a non-empty set $A$ is a partition iff

1. For all $X \in \mathcal{A}$, $X \neq \emptyset$.
2. If $X, Y \in \mathcal{A}$, then $X = Y$ or $X \cap Y = \emptyset$.
3. $\bigcup_{X \in \mathcal{A}} = A$. 
Construct the partitions of the following sets under the given equivalence relation.

1. \( \mathbb{Z} \mod 5 \).
2. \( \mathbb{Z} \mod 7 \).
3. \( \mathbb{R} \) under \( xRy \iff \lfloor y \rfloor = x \).
4. \( \mathbb{R}^2 \) under \((x, y)R(u, v) \iff x^2 + y^2 = u^2 + v^2 \).