SHOW YOUR WORK. Do not evaluate factorials or binomial coefficients.

1. Prove, using mathematical induction, that \(1 + 2 + \ldots + n = \frac{n(n+1)}{2}\). (Carefully separate Basis Step and Inductive Step, and indicate the inductive hypothesis explicitly).

2. Prove, using mathematical induction, that \(f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}\). (Carefully separate Basis Step and Inductive Step, and indicate the inductive hypothesis explicitly).
3. In how many ways can a photographer at a wedding arrange seven people (left to right), including the bride and the groom, if the bride and groom must be next to each other?

4. How many students, each of whom comes from one of the 50 states, must be enrolled at a university to guarantee that at least 23 students come from the same state?

5. How many license plates consisting of three letters followed by three decimal digits contain no repeated letters or digits?
6. What is the coefficient of $x^6$ in the simplified expansion of $(3x - 2y)^{10}$?

7. Seven women and nine men are on the faculty in a math department. How many ways are there to select a committee of five if the committee may not be all-male?

8. Consider the following recursive algorithm:

```plaintext
procedure foobar (n:positive integer)
if n = 1 then foobar(1):=2
else foobar(n):=3*foobar(n-1)
end
```

What value is calculated as “foobar(4)”?
9. How many different five-card poker hands have three of a kind, but not four of a kind, and not “full house”?

10. What is the probability that a five-card poker hand has exactly two diamonds and exactly two hearts?

11. Find a recurrence relation for $a_n$, the number of ternary strings of length $n$ having one or more instances of the substring ’22’. 