Editorial Note. The proposer gave indications of a solution as follows: The radical center Ω of the four spheres (O_1'), (O_2'), (O_3'), (O_4') coincides with the radical center of the second set of spheres on (O_1), (O_2), (O_3), (O_4); and Ω is the isogonal conjugate of the circumcenter O of $T=A_1A_2A_3A_4$ with respect to T. Also the points A_i' and A_i'' are the inverses of A_i in two inversions with the same pole Ω. From this it follows that the two tetrahedrons $T'=A_1'A_2'A_3'A_4'$ $T''=A_1''A_2''A_3''A_4''$ are homothetic, the center being Ω. (2). The circumcenters of T, T', $O_1'O_2'O_3'O_4'$ are collinear, *etc.* see V. Thébault, *Mathesis*, 1922, p. 363.

The proofs of these statements follow easily from theorems in Court's *Modern Pure Solid Geometry*, p. 246, 756, and p. 244, 752. Thus the three spheres, (O_2'), (O_3'), (O_4') are orthogonal to a sphere (Ω, r_1) with center Ω and radius r_1, intersect in A_1 and A_1' which are inverses with respect to this sphere, and hence $\Omega A_1 \cdot \Omega A_1' = r_1^2$. Similarly, $\Omega A_1 \cdot \Omega A_1'' = r_2^2$, and then $\Omega A_1'/\Omega A_1'' = r_1^2/r_2^2$, and the proof of (1) follows.

Since the circumspheres (T'), (T) are inverses in the first inversion, and (T''), (T) are inverses in the second, the three centers are collinear with Ω. We show next that the circumcenters of the tetrahedrons $O_1O_2O_3O_4$, $O_1'O_2'O_3'O_4'$ lie also on $O\Omega$ and this will conclude the proof of (2). Let N be the midpoint of $O\Omega$, and O_i' the center of (O_i'). Then $OO_i' = 2OO_i$, and N is the center of the common pedal sphere of the isogonal conjugates O and Ω, with respect to T, with the radius NO_i; and Ω is the center of the sphere $(O_1'O_2'O_3'O_4')$ with the radius $\Omega O_i' = 2NO_i$.

A Number Theory Function

4002 [1941, 483]. *Proposed by F. A. Lewis, University of Alabama*

Give an interpretation to the function that results from the Euler ϕ-function when the minus signs are changed to plus, namely $f(n) = n(1+1/p_1)(1+1/p_2) \cdots (1+1/p_k)$.

I. *Solution by E. P. Starke, Rutgers University*

Let $p_1, p_2, \cdots, p_k$ be the distinct prime divisors of n and set $m = n/p_1p_2 \cdots p_k$, where a_i is the exponent of p_i in n. Then $f(n)$ is the sum of all numbers which are simultaneously multiples of m and divisors of n. In other words: Let S be any divisor of n which is not divisible by a square, and let T be its complementary factor $(S\cdot T=n)$; then $f(n)$ is the sum of all numbers T. Of course, if $a_1 = a_2 = \cdots = a_k = 1$, $f(n)$ is the sum of all divisors of n. These statements follow immediately from two evident facts:

(1) $$f(p_i^{a_i}) = p_i^{a_i} + p_i^{a_i-1},$$

(2) $$f(xy) = f(x)\cdot f(y),$$

where x, y are relatively prime.

II. *Solution by the Proposer*

The required function is formed when $\phi_2(n)$ is divided by $\phi(n)$. Since $\phi_2(n)$

represents the number of elements of period n in an Abelian group of order n^2 and type (1, 1), the function formed represents the number of cyclic subgroups of order n in an Abelian group of order n^2 and type (1, 1).

Note by D. M. Seward, University of Tenn.

Referring to Dickson's *History of the Theory of Numbers*, Vol. 1, p. 123, we find: "R. Dedekind proved that, if n be decomposed in every way into a product ad, and if e is the g.c.d. of a, d, then

$$\sum a/e\phi(e) = n\prod (1 + 1/p) = f(n),$$

where a ranges over all divisors of n, and p over the prime divisors of n."

We shall use Jordan's generalization of Euler's ϕ-function, $J_k(n)$, p. 147. By definition, $J_k(n)$ is the number of different sets of k (equal or distinct) positive integers $\leqq n$, whose g.c.d. is prime to n. The formula is given

$$J_k(n) = n^k(1 - 1/p_1^k) \cdots (1 - 1/p_q^k).$$

We note that $f(n) = J_2(n)/\phi(n)$, $J_2(n) = \phi(n)\sum a/e\phi(e)$.

Letting $f_k(n) = n^k(1+1/p_1^k) \cdots (1+1/p_q^k)$, we have $f_k(n) = J_{2k}(n)/J_k(n)$.

It is noted (p. 150) that J. Vályi used $f(n)$ in his enumeration of the n-fold perspective polygons of n sides inscribed in a cubic curve.

Editorial Note. After the appearance of this problem in print the proposer discovered that his interpretation is given in Fricke's *Die Elliptischen Functionen*, vol. 2, p. 120. In the above mentioned volume of Dickson it is also stated on page 155 that G. A. Miller evaluated $J_k(m)$ by noting that it is the number of operators of period m in the abelian group with k independent generators of period m. This leads to the proposer's interpretation for $k=2$.

NEWS AND NOTICES

Readers are invited to contribute to the general interest of this department by sending news items to C. O. Oakley, Haverford College, Haverford, Pennsylvania.

Dr. H. N. Wright, formerly professor of mathematics at the College of the City of New York, was installed as president of the college on September 30, 1942. Professor H. F. MacNeish represented the Mathematical Association on this occasion.

Professor Virgil Snyder of Cornell University was one of four graduates of Iowa State College to receive the Chicago Merit Award in June 1942. The awards are presented by Chicago alumni of the college.