

Orthogonal Sets

- Determine which of the following sets are orthogonal using the inner product $\langle p(x), q(x) \rangle = p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1) + p(2)q(2)$.
 - $\{x^3 - 3x^2 + 2x, x^3 - x^2 - 2x, x^3 - 2x^2 - x + 2\}$
 - $\{1, x, x^2\}$
 - $\{x^3 - x + 1, 2x^3 - 3x^2 - 5x + 2, 2x^3 - 3x^2 + x + 2\}$
- Determine which of the following sets are orthogonal using the inner product $\langle f(x), g(x) \rangle = \int_{-\pi}^{\pi} f(x)g(x) dx$.
 - $\{1, x, x^2\}$
 - $\{1, \sin x, \cos x\}$
 - $\{1, x, x^2 - \frac{\pi^2}{3}\}$
- Normalize the vectors using the given inner product.
 - $p(x) = x$ using $\langle p(x), q(x) \rangle = p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1) + p(2)q(2)$.
 - $f(x) = 1$ using $\langle f(x), g(x) \rangle = \int_{-\pi}^{\pi} f(x)g(x) dx$.