# Math 231 Introduction to Discrete Mathematics
# Exam 1 Key

**Instructions**

1. Do NOT write your answers on these sheets. Nothing written on the test papers will be graded.
2. Please begin each section of questions on a new sheet of paper.
3. Please do not write answers side by side.
4. Please do not staple your test papers together.
5. Limited credit will be given for incomplete or incorrect justification.

**Questions**

1. Sets (2 each)
   Use the data in Figure 1.
   Let $UAA$ be the set of employees of UAA (the entire table).
   Let $A$ be the set of employees at the Anchorage campus.
   Let $KPC$ be the set of employees at the Kenai campus.
   Let $MatSu$ be the set of employees at the MatSu campus.
   Let $UNAC$ be the set of employees in UNAC.
   Let $UAFT$ be the set of employees in UAFT.
   Let $Upper$ be the set of employees teaching upper division courses (upper=yes).
   Sets may be listed using just the distinct portion of the ID.

   (a) Find $A \cap UNAC$.

   $$\{20100001, 20100002, 20100003, 20100004\}$$

   (b) Find $A \cup KPC$.

   $$\{20100001, 20100002, 20100003, 20100004, 20100005, 20100006, 20100007, 20100008\}$$

   (c) Find $A - UAFT$.

   $$A - UAFT = A \cap UNAC.$$

   (d) Find $\overline{UNAC}$.

   $$\{20100005, 20100007, 20100008, 20100009, 20100010, 20100011\}$$

2. Counts (2 each)

   (a) If you have 14 choices for ice cream, 30 choices for items to mix add to the ice cream, and you order two, distinct items to add, how many options do you have?

   $$\boxed{14}\boxed{30}\boxed{29} = 14(30)(29).$$

   To ignore the order of choosing the two items

   $$\frac{(14)(30)(29)}{2}.$$

   (b) If passwords were only 8 characters long consisting of only the lower and upper case letters of the English alphabet and the 10 numerical digits, how many possible passwords can be constructed?

   $$\boxed{62}\boxed{62}\boxed{62}\boxed{62}\boxed{62}\boxed{62}\boxed{62}\boxed{62} = 62^8.$$

   (c) How many ways can you line up the members of $A$ to have their cholesterol measured?

   $$\boxed{5}\boxed{4}\boxed{3}\boxed{2}\boxed{1} = 5!$$

3. Logic (4 each)

   (a) Re-write with negatives only at the lowest level. $\neg(\forall x UNAC(x) \rightarrow (A(x) \wedge Upper(x)))$

$$\begin{aligned} \neg(\forall x UNAC(x) \rightarrow (A(x) \wedge Upper(x)) &\equiv \\ \exists x UNAC(x) \ni \neg(A(x) \wedge Upper(x)) &\equiv \\ \exists x UNAC(x) \ni (\neg A(x) \vee \neg Upper(x)) & \end{aligned}$$

   (b) Is the statement true?
   Yes, there is a UNAC faculty member not at the Anchorage campus.

   (c) When is the following statement true? List combinations of $A, B, P$ the make the statement true.
   ((KIAS < TargetSpeed) and (Power > PowerCurveMax)) or ((KIAS > TargetSpeed) and (Power < PowerCurveMax)) implies PowerNeeded
   $((A \wedge \neg B) \vee (\neg A \wedge B)) \rightarrow P$

| $A$ | $B$ | $P$ | $\neg A$ | $\neg B$ | $A \wedge \neg B$ | $\neg A \wedge B$ | $\vee$ | $\rightarrow$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |

   All cases except when $A = 0$, $B = 1$, $P = 0$ and $A = 1$, $B = 0$, and $P = 0$.

   (d) Write a boolean algebra expression for the function in Figure 2.

$$b(x, y, z) = \overline{xyz} + \overline{x}y\overline{z} + x\overline{yz}$$

   (e) Why is the following test useless? if $\neg x \vee x \vee (x \wedge y)$

$$\begin{aligned} \neg x \vee x \vee (x \wedge y) &\equiv \\ 1 \vee (x \wedge y) &\equiv 1 \end{aligned}$$

   Thus this is always true.

4. Proof (5 each)

   (a) Prove: If $A \subset B$ then $A \cap B = A$.
   Proof: First, $A \cap B \subset A$. Let $x \in A \cap B$. Thus $x \in A$ and $x \in B$. Thus $x \in A$, so $A \cap B \subset A$.
   Second, $A \subset A \cap B$. Let $x \in A$. Because $A \subset B$, $x \in B$ as well. Thus $x \in A$ and $x \in B$, or $x \in A \cap B$. Thus $A \subset A \cap B$.
   Because $A \cap B \subset A$ and $A \subset A \cap B$, $A \cap B = A$. □

| ID | Campus | Union | Department | Upper |
|---|---|---|---|---|
| 20100001 | Anchorage | UNAC | Philosophy | Yes |
| 20100002 | Anchorage | UNAC | Math & Stats | Yes |
| 20100003 | Anchorage | UNAC | Math & Stats | No |
| 20100004 | Anchorage | UNAC | Biology | Yes |
| 20100005 | Anchorage | UAFT | English | Yes |
| 20100006 | Kenai | UNAC | Biology | Yes |
| 20100007 | Kenai | UAFT | Math & Stats | Yes |
| 20100008 | Kenai | UAFT | English | No |
| 20100009 | MatSu | UAFT | Math & Stats | No |
| 20100010 | MatSu | UAFT | English | No |
| 20100011 | MatSu | UAFT | Veterinary Tech | No |

Figure 1: Database

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| y | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| z | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| w | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |

Figure 2: Boolean Function: $b(x, y, z) = w$