Assignment 3

The arclength of a space curve is given by the integral

\[ \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt \]

The surface area of revolution obtained by rotating about the y-axis is

\[ \int_a^b 2\pi x \sqrt{1 + (f'(x))^2} \, dx. \]

The volume under the surface of revolution about the y-axis is

\[ \int_a^b 2\pi x f(x) \, dx. \]

1. Consider the spacial curve \( x = t, \ y = \sin(nt), \ z = \cos(nt) \) for \( t \in [0, 10\pi/n] \).
   (a) Plot this curve for \( n = 1, 2, 3 \).
   (b) Produce an animation of this curve for \( n \) from 1 to 5. Note you will need to use some plot options to make this easy to view.
   (c) Calculate the arclength of this curve. Leave \( n \) as a variable when calculating.
   (d) Evaluate the length of the curve for specific \( n \) values 1, 2, 3, 4, 5.
   (e) Calculate the limit of the arclength as \( n \) approaches \( \infty \).
   (f) If this were a spring, would it make sense for the arclength to change as the spring stretches?

2. Consider the 3D surface obtained by rotating \( f_n(x) = \sin(nx) + 1, x \in [0, 2\pi] \) about the y-axis.
   (a) Plot the surface of revolution for \( n = 1, 2, 3 \).
   (b) Produce an animation of this curve for \( n \) from 1 to 5.
   (c) Calculate the surface area of this curve for \( n \) from 1 to 5.
   (d) Conjecture, based on the above experiment, what is happening to the area as \( n \) approaches infinity

3. Again consider the 3D surface obtained by rotating \( f_n(x) = \sin(nx) + 1, x \in [0, 2\pi] \) about the y-axis.
   (a) Calculate the volume beneath this curve for \( n \) from 1 to 5.
   (b) Calculate the volume beneath this curve for arbitrary \( n \).
   (c) Evaluate the limit of the volume beneath the curve as \( n \) approaches infinity.
   (d) What does surface seem to become as \( n \) approaches infinity? Your answer should be geometric.
   (e) Does your answer above explain the limit of the volume?