Math 205
Test 3 Key

Instructions

1. Do NOT write your answers on these sheets. Nothing written on the test papers will be graded.
2. Please begin each section of questions on a new sheet of paper.
3. Please do not write answers side by side.
4. Please do not staple your test papers together.
5. Limited credit will be given for incomplete or incorrect justification.

Questions

1. Angles (2 per part)

   (a) As the rays in Figure 1 spread further apart does the angle become smaller or larger?
   The angle becomes larger.

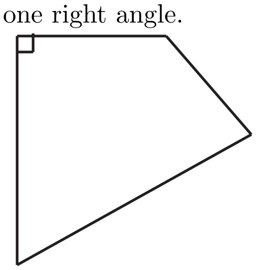
   (b) Figure 2 shows a quadrilateral. Note that both side angles have fixed measure 60 degrees. The top two sides should be extended to make them meet. What is the top angle if the bottom angle is 30°, 60°, 90°, 120°?

   The sum of the degrees of a rectangle is 360°. Thus

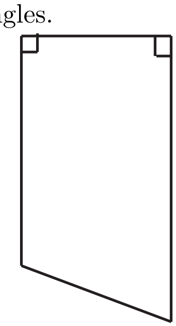
30°	$360^\circ - 2(60^\circ) - 30^\circ = 210^\circ$
60°	$360^\circ - 2(60^\circ) - 60^\circ = 180^\circ$
90°	$360^\circ - 2(60^\circ) - 90^\circ = 150^\circ$
120°	$360^\circ - 2(60^\circ) - 120^\circ = 120^\circ$

   (c) As the bottom angle increases, does the top angle increase or decrease? Why?
   As the bottom angle increases the top angle decreases, because the sum of the angles remains constant (the number of sides is not changing).

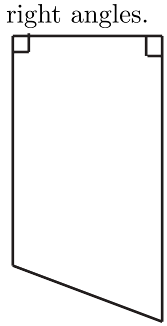
   (d) Draw a quadrilateral with **exactly** one right angle.



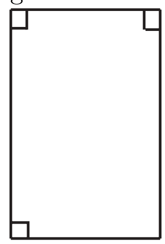
   (e) Draw a quadrilateral with two right angles.



   (f) Draw a quadrilateral with **exactly** two right angles.



(g) Draw a quadrilateral with three right angles.



(h) Is it possible to draw a quadrilateral with **exactly** three right angles? If not, why? If so, draw it.

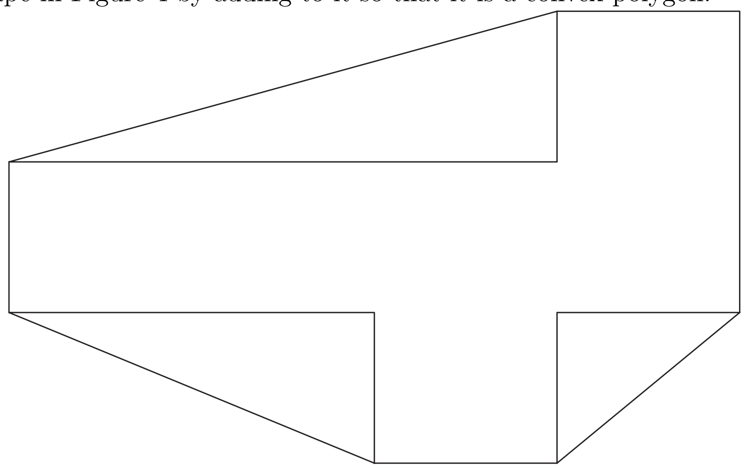
It is not possible. The sum of the angles of a quadrilaterl is $360°$. Thus if three angles are $90°$ the remaining angle must be $360° - 3(90°) = 90°$.

2. Shapes (2 per part)

(a) Determine if the shapes in Figure 3 are convex or not.

Part A	Not convex
Part B	Convex
Part C	Convex
Part D	Not convex

(b) Modify the shape in Figure 4 by adding to it so that it is a convex polygon.



(c) Figure 5 shows the way Günter labeled the angles of a polygon. He claims it is regular, because all the side lengths are the same and all the angles are the same. What do you need to explain to Günter to correct his mistake?

Explain to Günter that he needs to measure all interior or all exterior angles. He has used some of both.

(d) How does this relate to the Jordan Curve theorem?

The Jordan Curve theorem states that every simply, closed curve divides the plane into three parts: inside, boundary, and outside. Günter needs to look at angles in only one region.

(e) Draw a figure with three sides of the same length.

(f) Is it possible to draw a three sided figure with equal sides but unequal angles?

No, it is not.

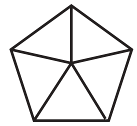
(g) Draw a figure with four sides of the same length.

(h) Is it possible to draw a four sided figure with equal sides but unequal angles? If not, explain why. If so, draw an example.

It is. Non-square rhombi are such shapes.

3. Kicking It Up a Dimension

(a) (1) Draw a regular pentagon.



(b) (2) Draw a point at the center of the pentagon. Draw lines from the center to the vertices. Into what shapes have you divided the pentagon?

The pentagon has been divided up into triangles.

(c) (2) Consider the regular cube. Note the lines from the center to the vertices. Into what shapes do these lines divide the cube?

The resulting shapes are square pyramids.

(d) (2) Consider the regular icosahedron. Note the lines from the center to the vertices. Into what shapes do these lines divide the icosahedron?

The resulting shapes are triangular pyramids (regular tetrahedra).

(e) (3) Calculate the perimeter of the rectangle in Figure 8.

The perimeter is

$$3 + 2 + 3 + 2 = 10.$$

(f) (3) Calculate the area of the rectangle in Figure 8.

The area of the rectangle is

$$(3)(2) = 6.$$

(g) (3) Calculate the surface area of the rectangular box in Figure 9.

The surface are consists of two rectangles of area $(4)(3) = 12$, two rectangles of area $(4)(2) = 8$, and two rectangles of area $(3)(2) = 6$. Thus the total surface area is

$$(2)(12) + (2)(8) + (2)(6) = 52.$$

(h) (3) Calculate the volume of the rectangular box in Figure 9.

The volume is

$$(2)(3)(4) = 24.$$

(i) (2) Perimeter is to area as what is to volume? Why?

Perimeter is to area as **surface area** is to volume, because perimeter and surface area measure the boundary of an object. The boundary is one dimension less than the object.

4. Happy and Cross Sections

   (a) Consider the sections of a right circular cone in Figure .

      i. (1) What shape is produced by the vertical cross section?
      A triangle is the vertical cross section.

      ii. (1) What shape is produced by the horizontal cross section?
      A circle is the horizontal cross section.

      iii. (4) Why are the sections in Parts C and D different from those in Parts A and B?
      The cross sections are neither vertical nor horizontal, they are skewed cross sections.

   (b) (2) If the set of faces in Part A of Figure is truncated, how many sides will the resulting side have?

   The truncation will cut five faces, so there will be 5 sides.

   (c) (2) If the set of faces in Part B of Figure is truncated, how many sides will the resulting side have?

   The truncation will cut three faces, so there will be 3 sides.

   (d) (2) List shapes of all the sides if the set of faces in Part C of Figure is truncated.

   The truncation will cut three faces, so the new sides will have 3 sides. The old faces had three sides and will have three added producing six sides. The shapes are **triangles** and **hexagons**.

   (e) (2) The tetrahedron (object in Part C of Figure) has four sides. After truncating all the vertices, how many sides will the resulting object have?

   There are four vertices to truncate. This will add four faces. The result is eight faces.

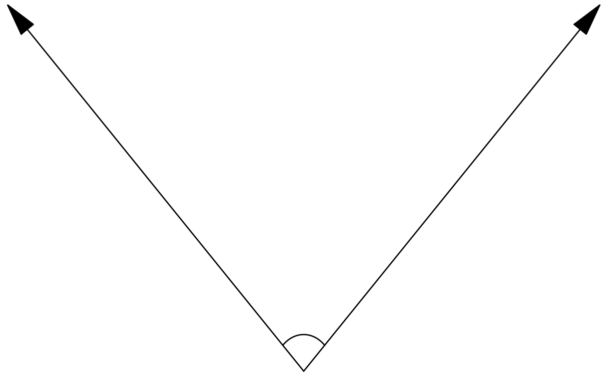


Figure 1: Angle between rays

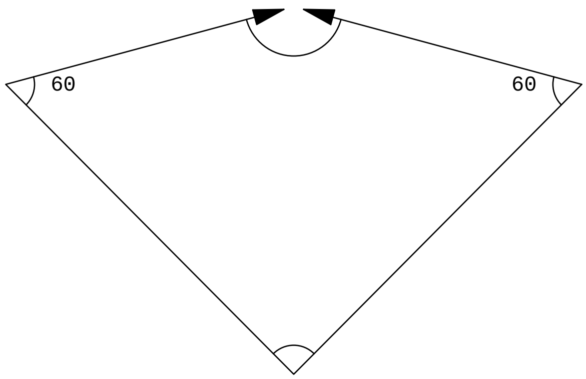


Figure 2: Angles of Quadrilaterals

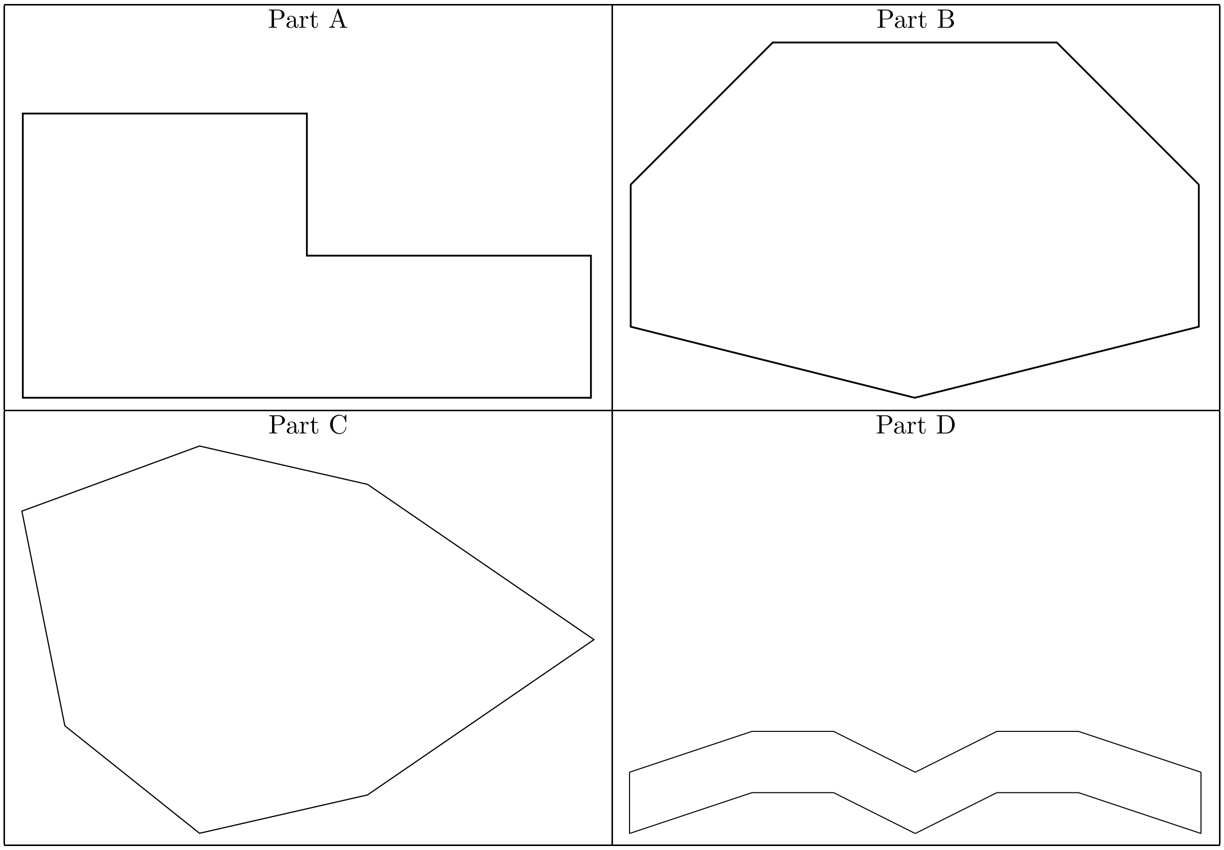


Figure 3: Assorted Polygons

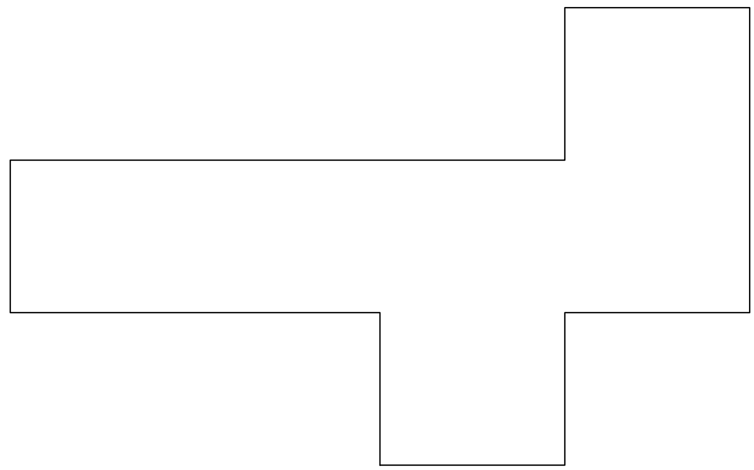


Figure 4: Irregular Polygon

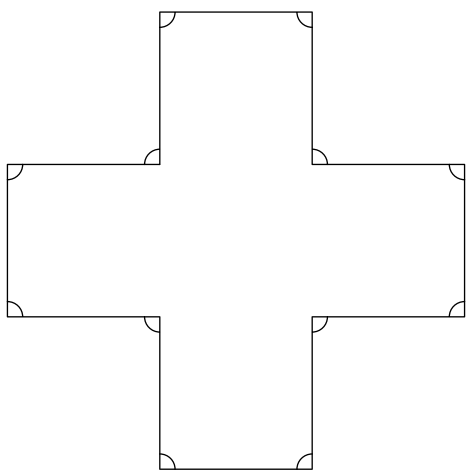


Figure 5: Non-Regular Polygon

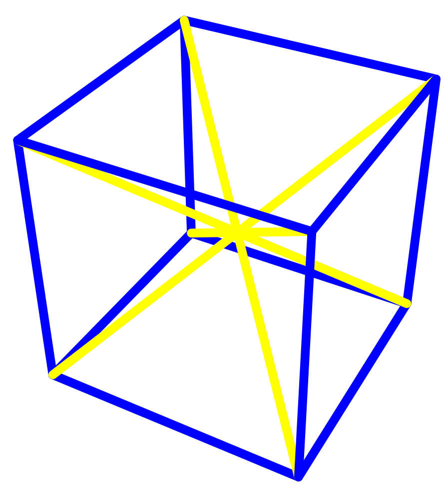


Figure 6: Cube with Struts

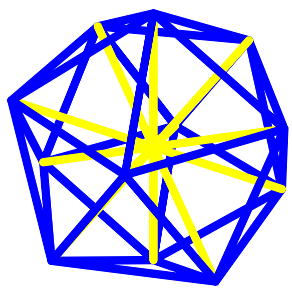


Figure 7: Icosahedron with Struts

Figure 8: Rectangle

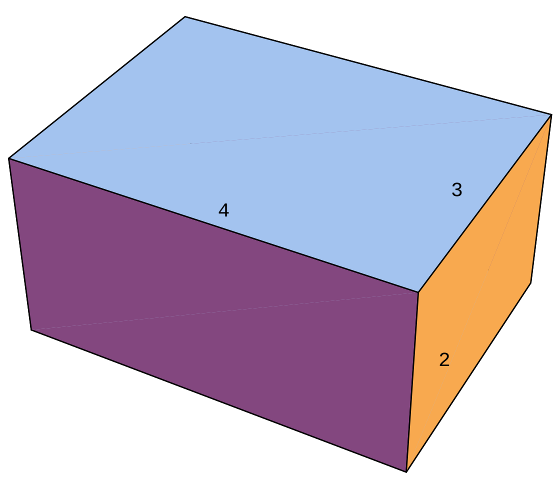


Figure 9: Box

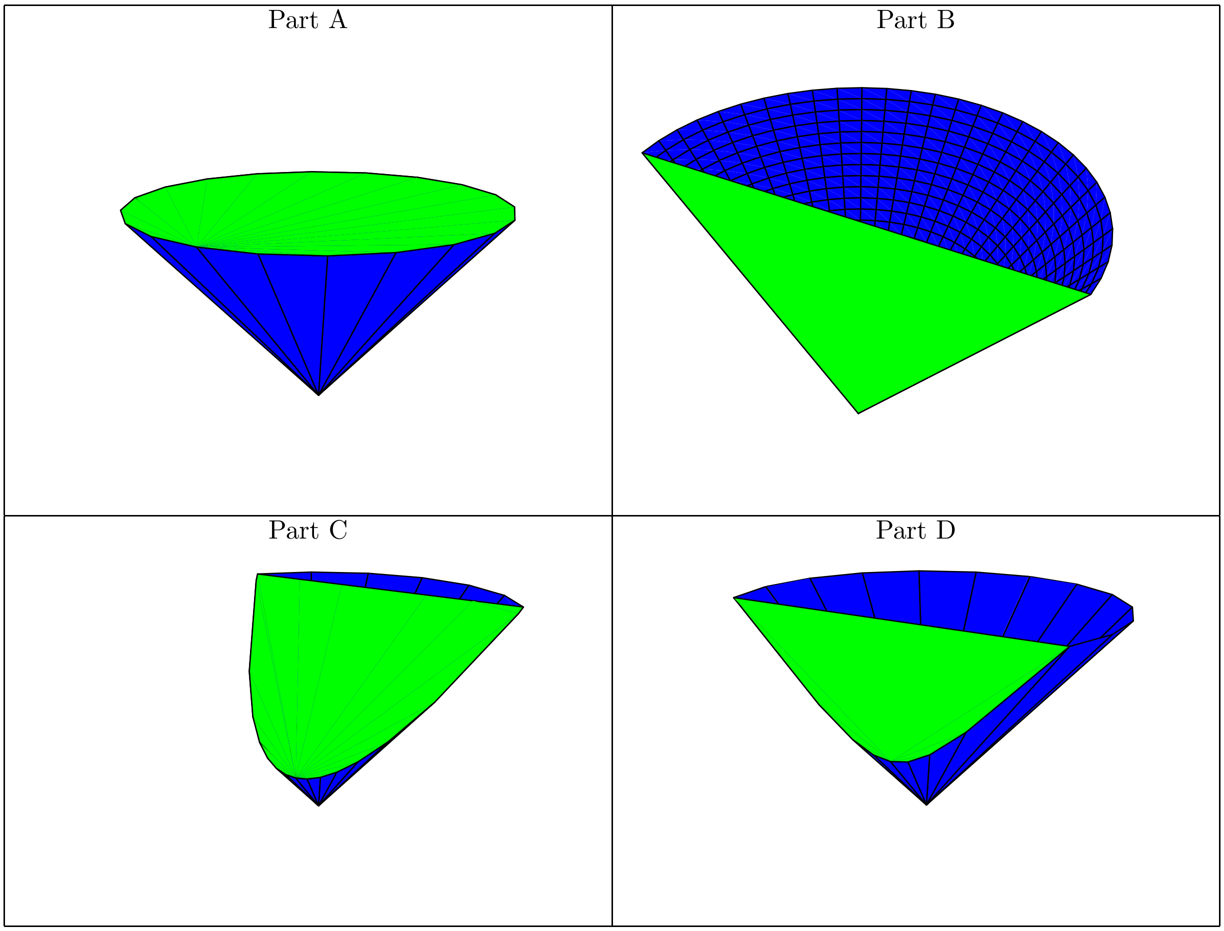


Figure 10: Sections of Cones

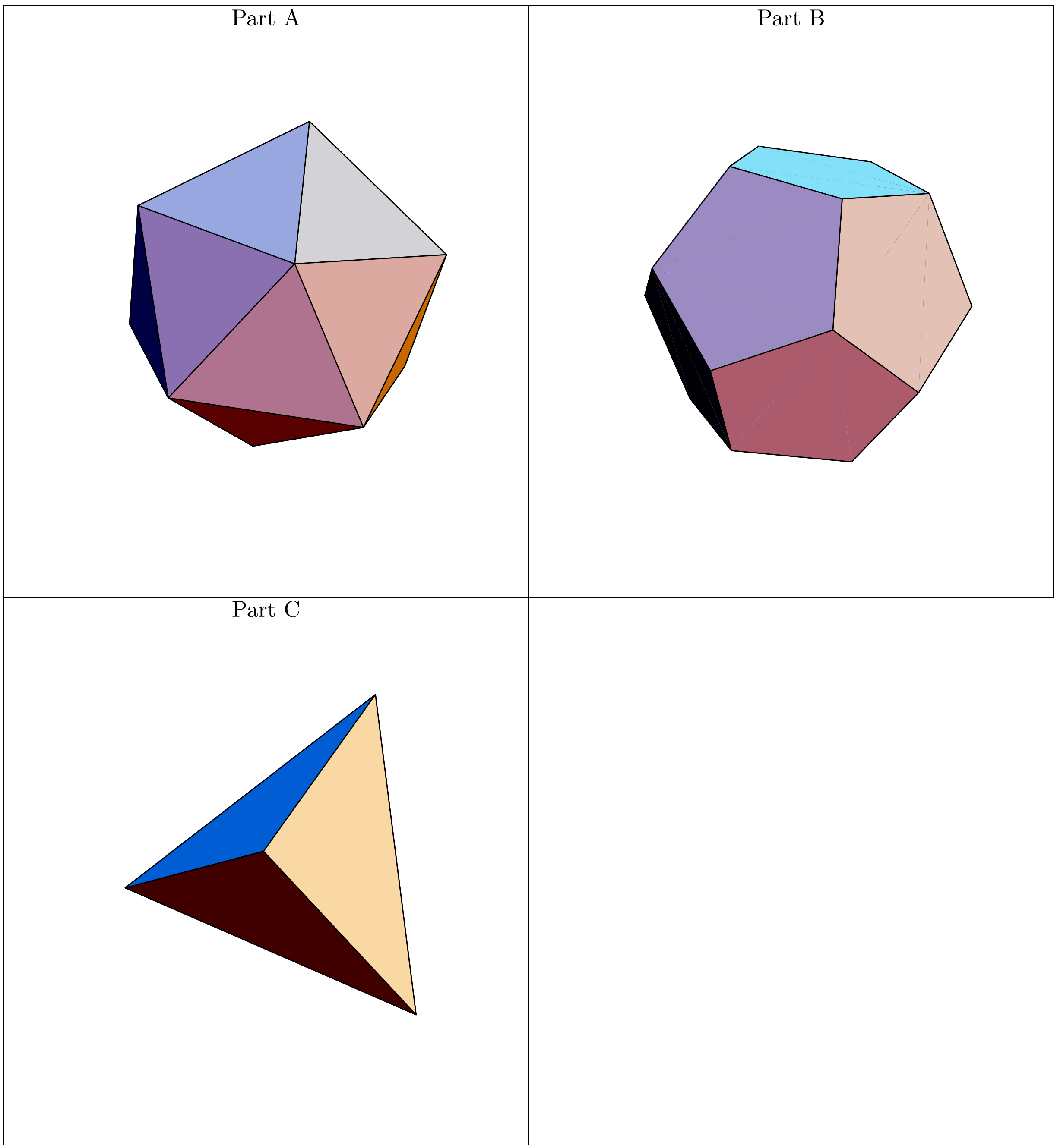


Figure 11: Polyhedra