# Angles and Area of Polygons

1. Central Angles of Regular Polygons

   (a) Take a copy of the polygons in circles. For each polygon use a straightedge to draw lines from the center to each vertex. These lines will look like spokes of a wheel.

   (b) Note that each polygon has been divided up into triangles. Look at the inscribed square. Inscribed means it is drawn inside a circle with the vertices touching the circle. It is divided up into four triangles by the lines that you drew. What do two adjacent triangles share?

   (c) For any two of the four triangles what is true about their sides?

   (d) A theorem of geometry called Side-Side-Side states that two triangles are congruent if the three sides have the same lengths. What is true then about any two angles formed by the spokes?

   (e) What is the sum of all these central angles?

   (f) What is the measure of one of these central angles?

   (g) Look at the other inscribed polygons. Calculate their central angles.

   (h) If a polygon has $n$ sides, what is the measure of its central angles?

2. Measuring Angles in Regular Polygons

   (a) As the number of sides increases, do the central angles become smaller or larger?

   (b) As the number of sides increases, do the vertex angles appear to become smaller or larger?

   (c) Consider again the inscribed square. What is true about the two sides of any one triangle in the square?

   (d) A triangle with two sides of the same length is called iscoceles. The two angles opposite the equal sides have the same measure. The sum of the angles of any triangle is 180 degrees. What is the sum of the two equal angles in the triangle?

   (e) What is the relationship between the vertex angle of the square and some of the angles of the isosceles triangles?

   (f) What is the vertex angle of the square?

   (g) Repeat these steps for the other polygons.

   (h) What is the measure of a vertex angle of a polygon with $n$ sides?

3. Area of Regular Polygons

   (a) Note that the area of an isosceles triangle can be computed by Area $= \frac{r^2}{2} \sin \alpha$. Where $\alpha$ is the central angle. What is the area of one of the triangles from the inscribed square? Suppose that the radius of the circles is 1.

   (b) What is the area of one of the triangles from the other inscribed polygons?

   (c) What is the area of one of the triangles from an inscribed polygon with $n$ sides?

   (d) What is the area of an inscribed polygon with $n$ sides?

4. An Amazing Conclusion

   (a) Using the formula you obtained above, compute the area for regular polygons with 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15 sides. You may want to use a calculator.

   (b) Write the list of areas in a table in order of increasing sides.

   (c) Take an unused copy of the polygon sheets and color in the inside of each polygon. Notice that the radius of each circle is the same. Suppose the radius has length 1.

   (d) Consider the unshaded region between the polygons and the circles. As the number of sides of the polygons increases, what happens to this unshaded area?

   (e) We will rephrase your answer to the previous question. As the number of sides of the polygons increases what happens to the area of the polygons?

   (f) Based on these examples would the area of the polygons become infinitely large as the number of sides increases or would the area have a bound (limitation)?

   (g) We will rephrase the answer again. As the number of sides of the polygons increases what happens to the area of the polygons with respect to the circle?

   (h) Obtain a decimal estimate for the number $\pi$ using your results above. Is your estimate particularly close? You may want to use a calculator.

5. Your Conclusion: Write a substantial paragraph or two about what you have learned through this set of exercises.