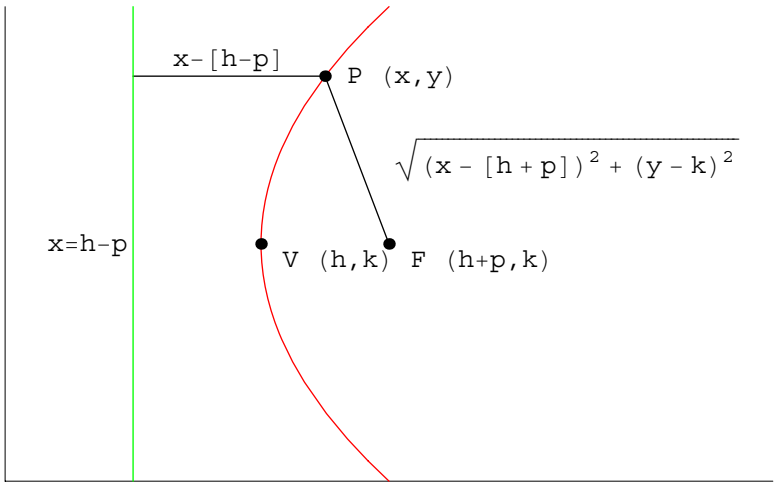
Discovering Parabolas

Definition

A set of points is a *parabola* if and only if
the distance from any point P on the parabola to a fixed point F, called the focus
equals
the distance from that point P to a fixed line l, called the directrix.

Diagram



Derivation

$$
\begin{aligned}
x-[h-p] &= \sqrt{[x-(h+p)]^2+(y-k)^2}. \\
(x-h+p)^2 &= [x-h-p]^2+(y-k)^2. \\
x^2-2hx-2hp+h^2+2px+p^2 &= x^2-2hx+2ph+h^2-2px+p^2+(y-k)^2. \\
4px-4ph &= (y-k)^2. \\
4p(x-h) &= (y-k)^2.
\end{aligned}
$$