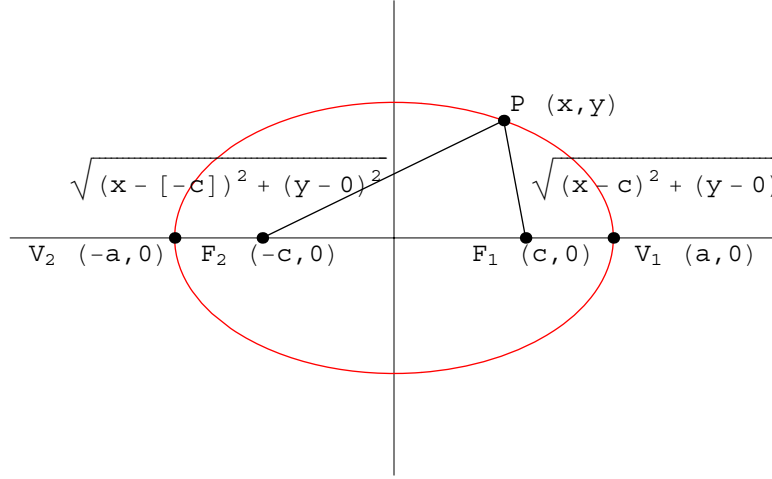


Discovering Ellipses

Definition

A set of points is a *ellipse* if and only if the sum of the distance from any point P on the parabola to a fixed point F_1 , called the focus and the distance from that point P to a second focus F_2 is constant.

Diagram



Derivation

$$\begin{aligned}
 \sqrt{[x - (-c)]^2 + y^2} + \sqrt{(x - c)^2 + y^2} &= 2a. \\
 \sqrt{(x + c)^2 + y^2} &= 2a - \sqrt{(x - c)^2 + y^2}. \\
 (\sqrt{(x + c)^2 + y^2})^2 &= (2a - \sqrt{(x - c)^2 + y^2})^2. \\
 (x + c)^2 + y^2 &= 4a^2 - 4a\sqrt{(x - c)^2 + y^2} + [(x - c)^2 + y^2] \\
 x^2 + 2cx + c^2 + y^2 &= 4a^2 - 4a\sqrt{(x - c)^2 + y^2} + x^2 - 2cx + c^2 + y^2. \\
 4cx &= 4a^2 - 4a\sqrt{(x - c)^2 + y^2}. \\
 cx &= a^2 - a\sqrt{(x - c)^2 + y^2}. \\
 cx - a^2 &= -a\sqrt{(x - c)^2 + y^2}. \\
 (cx - a^2) &= (-a\sqrt{(x - c)^2 + y^2})^2. \\
 c^2x^2 - 2a^2cx + a^4 &= a^2[(x - c)^2 + y^2]. \\
 c^2x^2 - 2a^2cx + a^4 &= a^2(x^2 - 2cx + c^2 + y^2). \\
 c^2x^2 - 2a^2cx + a^4 &= a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2. \\
 c^2x^2 + a^4 &= a^2x^2 + a^2c^2 + a^2y^2. \\
 a^4 - a^2c^2 &= a^2x^2 - c^2x^2 + a^2y^2. \\
 a^2(a^2 - c^2) &= (a^2 - c^2)x^2 + a^2y^2. \\
 a^2b^2 &= b^2x^2 + a^2y^2. \\
 1 &= \frac{x^2}{a^2} + \frac{y^2}{b^2}.
 \end{aligned}$$