A New Method

$$\begin{array}{rrrl} x & -3y & +2z= & 8. \\ 2x & -y & +z= & 10. \\ 4x & +2y & -3z= & 10. \end{array}$$

Systems of linear equations (like that above) can be solved using the elimination method.

A New Method

$$\begin{array}{rrrl} x & -3y & +2z= & 8. \\ 2x & -y & +z= & 10. \\ 4x & +2y & -3z= & 10. \end{array}$$

Systems of linear equations (like that above) can be solved using the elimination method. To be efficient using the elimination method we need a new notation. On the following pages we use the elimination method, and we show the new notation side-by-side.

Matrix Notation

$$
\begin{array}{rrrll}
x & -3y & +2z= & 8. & E1 \\
2x & -y & +z= & 10. & E2 \\
4x & +2y & -3z= & 10. & E3
\end{array}
\qquad
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
2 & -1 & 1 & 10 \\
4 & 2 & -3 & 10
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
 x & -3y & +2z = & 8. & E1 \\
2x & -y & +z = & 10. & E2 \\
4x & +2y & -3z = & 10. & E3
\end{array}
\qquad
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
2 & -1 & 1 & 10 \\
4 & 2 & -3 & 10
\end{array}\right]
$$

$$
\begin{array}{rrrrrl}
-2(& x & -3y & +2z) = & -2(8). & E1 \\
 & -2x & +6y & -4z = & -16. & \\
+ & 2x & -y & +z = & 10. & E2 \\
\hline
 & & 5y & -3z = & -6. & E4
\end{array}
$$

Matrix Notation

$$
\begin{array}{rrrrl}
 & x & -3y & +2z= & 8. \quad E1 \\
 & 2x & -y & +z= & 10. \quad E2 \\
 & 4x & +2y & -3z= & 10. \quad E3
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 2 & -1 & 1 & 10 \\ 4 & 2 & -3 & 10 \end{array}\right]
\qquad R_2 \leftarrow -2R_1 + R_2
$$

$$
\begin{array}{rrrrl}
-2(& x & -3y & +2z)= & -2(8). \quad E1 \\
 & -2x & +6y & -4z= & -16. \\
+ & 2x & -y & +z= & 10. \quad E2 \\
\hline
 & & 5y & -3z= & -6. \quad E4
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 4 & 2 & -3 & 10 \end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
 & x & -3y & +2z = & 8. \quad E1 \\
 & 2x & -y & +z = & 10. \quad E2 \\
 & 4x & +2y & -3z = & 10. \quad E3
\end{array}
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
2 & -1 & 1 & 10 \\
4 & 2 & -3 & 10
\end{array}\right]
\quad R_2 \leftarrow -2R_1 + R_2
$$

$$
\begin{array}{rrrrl}
-2(& x & -3y & +2z) = -2(8). & E1 \\
 & -2x & +6y & -4z = -16. & \\
+ & 2x & -y & +z = 10. & E2 \\
\hline
 & & 5y & -3z = -6. & E4
\end{array}
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & -3 & -6 \\
4 & 2 & -3 & 10
\end{array}\right]
$$

$$
\begin{array}{rrrrl}
-4(& x & -3y & +2z) = -4(8). & E1 \\
 & -4x & +12y & -8z = -32. & \\
+ & 4x & +2y & -3z = 10. & E3 \\
\hline
 & & 14y & -11z = -22. & E5
\end{array}
$$

Matrix Notation

$$
\begin{array}{rrrrl}
 & x & -3y & +2z= & 8. \quad E1 \\
 & 2x & -y & +z= & 10. \quad E2 \\
 & 4x & +2y & -3z= & 10. \quad E3
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 2 & -1 & 1 & 10 \\ 4 & 2 & -3 & 10 \end{array}\right]
\qquad R_2 \leftarrow -2R_1 + R_2
$$

$$
\begin{array}{rrrrl}
-2(& x & -3y & +2z)= & -2(8). \quad E1 \\
 & -2x & +6y & -4z= & -16. \\
+ & 2x & -y & +z= & 10. \quad E2 \\
\hline
 & & 5y & -3z= & -6. \quad E4
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 4 & 2 & -3 & 10 \end{array}\right]
\qquad R_3 \leftarrow -4R_1 + R_3
$$

$$
\begin{array}{rrrrl}
-4(& x & -3y & +2z)= & -4(8). \quad E1 \\
 & -4x & +12y & -8z= & -32. \\
+ & 4x & +2y & -3z= & 10. \quad E3 \\
\hline
 & & 14y & -11z= & -22. \quad E5
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 14 & -11 & -22 \end{array}\right]
$$

Matrix Notation

$$\begin{aligned} 5y &\quad -3z = -6. \quad E4 \\ 14y &\quad -11z = -22. \quad E5 \end{aligned}$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 14 & -11 & -22 \end{array}\right]$$

Matrix Notation

$$\begin{aligned} 5y &- 3z = -6. && E4 \\ 14y &- 11z = -22. && E5 \end{aligned}$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 14 & -11 & -22 \end{array}\right]$$

$$\begin{aligned} -\tfrac{14}{5}(\quad 5y &- 3z) = -\tfrac{14}{5}(-6). && E4 \\ -14y &+ \tfrac{42}{5}z = \tfrac{84}{5}. && \\ + \quad 14y &- 11z = -22. && E5 \\ \hline -\tfrac{13}{5}z &= -\tfrac{26}{5}. && E6 \end{aligned}$$

Matrix Notation

$$\begin{array}{rrrrl} & 5y & -3z= & -6. & E4 \\ & 14y & -11z= & -22. & E5 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 14 & -11 & -22 \end{array}\right] \quad R_3 \leftarrow -\tfrac{14}{5}R_2 + R_3$$

$$\begin{array}{rrrrl} -\frac{14}{5}(& 5y & -3z)= & -\frac{14}{5}(-6). & E4 \\ & -14y & +\frac{42}{5}z= & \frac{84}{5}. & \\ + & 14y & -11z= & -22. & E5 \\ \hline & & -\frac{13}{5}z= & -\frac{26}{5}. & E6 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 0 & -\frac{13}{5} & -\frac{26}{5} \end{array}\right]$$

Matrix Notation

$$\begin{array}{rrrrl} & 5y & -3z= & -6. & E4 \\ & 14y & -11z= & -22. & E5 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 14 & -11 & -22 \end{array}\right] \quad R_3 \leftarrow -\tfrac{14}{5}R_2 + R_3$$

$$\begin{array}{rrrrl} -\frac{14}{5}(& 5y & -3z)= & -\frac{14}{5}(-6). & E4 \\ & -14y & +\frac{42}{5}z= & \frac{84}{5}. & \\ + & 14y & -11z= & -22. & E5 \\ \hline & & -\frac{13}{5}z= & -\frac{26}{5}. & E6 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 0 & -\frac{13}{5} & -\frac{26}{5} \end{array}\right]$$

$$\begin{array}{rrrrl} & & -\frac{13}{5}z= & -\frac{26}{5}. & E6 \\ & & z= & 2. & \end{array}$$

Matrix Notation

$$
\begin{array}{rrrrl}
 & 5y & -3z= & -6. & E4 \\
 & 14y & -11z= & -22. & E5
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 14 & -11 & -22 \end{array}\right]
\quad R_3 \leftarrow -\tfrac{14}{5}R_2 + R_3
$$

$$
\begin{array}{rrrrl}
-\frac{14}{5}(& 5y & -3z)= & -\frac{14}{5}(-6). & E4 \\
 & -14y & +\frac{42}{5}z= & \frac{84}{5}. & \\
+ & 14y & -11z= & -22. & E5 \\
\hline
 & & -\frac{13}{5}z= & -\frac{26}{5}. & E6
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 0 & -\frac{13}{5} & -\frac{26}{5} \end{array}\right]
\quad R_3 \leftarrow -\tfrac{5}{13}R_3
$$

$$
\begin{array}{rrrl}
 & -\frac{13}{5}z= & -\frac{26}{5}. & E6 \\
 & z= & 2. &
\end{array}
\qquad
\left[\begin{array}{rrr|r} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 0 & 1 & 2 \end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
x & -3y & +2z= & 8. & E1 \\
 & 5y & -3z= & -6. & E4 \\
 & & z= & 2. & E6
\end{array}
\qquad
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & -3 & -6 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
x & -3y & +2z= & 8. & E1 \\
 & 5y & -3z= & -6. & E4 \\
 & & z= & 2. & E6 \\
3(& & z)= & 3(2). & E6 \\
 & & 3z= & 6. & \\
 & 5y & -3z= & -6. & E4 \\
\hline
 & 5y & = & 0. & E7
\end{array}
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & -3 & -6 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
x & -3y & +2z= & 8. & E1 \\
 & 5y & -3z= & -6. & E4 \\
 & & z= & 2. & E6 \\
3(& & z)= & 3(2). & E6 \\
 & & 3z= & 6. & \\
 & 5y & -3z= & -6. & E4 \\
\hline
 & 5y & = & 0. & E7
\end{array}
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & -3 & -6 \\
0 & 0 & 1 & 2
\end{array}\right]
\qquad R_2 \leftarrow 3R_3 + R_2
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & 0 & 0 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
x & -3y & +2z= & 8. & E1 \\
 & 5y & -3z= & -6. & E4 \\
 & & z= & 2. & E6 \\
3(& & z)= & 3(2). & E6 \\
 & & 3z= & 6. & \\
 & 5y & -3z= & -6. & E4 \\
\hline
 & 5y & = & 0. & E7 \\
 & & & & \\
 & \frac{1}{5}(5y &) = & \frac{1}{5}(0). & E7 \\
 & y & = & 0. &
\end{array}
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & -3 & -6 \\
0 & 0 & 1 & 2
\end{array}\right]
\qquad R_2 \leftarrow 3R_3 + R_2
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 5 & 0 & 0 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
x & -3y & +2z= & 8. & E1 \\
 & 5y & -3z= & -6. & E4 \\
 & & z= & 2. & E6 \\
3(& & z)= & 3(2). & E6 \\
 & & 3z= & 6. & \\
 & 5y & -3z= & -6. & E4 \\
\hline
 & 5y & = & 0. & E7 \\
 & \frac{1}{5}(5y &) = & \frac{1}{5}(0). & E7 \\
 & y & = & 0. &
\end{array}
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 5 & -3 & -6 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_2 \leftarrow 3R_3 + R_2
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_2 \leftarrow \frac{1}{5}R_2
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrll}
x & -3y & +2z{=}8. & E1 & \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \\
 & y & {=}0. & E7 & \\
 & & z{=}2. & E6 &
\end{array}
$$

Matrix Notation

$$
\begin{array}{rrrrl}
 & x & -3y & +2z=8. & E1 \\
 & & y & =0. & E7 \\
 & & & z=2. & E6 \\
-2(& & & z)=-2(2). & E6 \\
 & & -2z= & -4. & \\
 & x & -3y \quad 2z= & 8. & E1 \\
\hline
 & x & -3y \quad = & 4. & E8
\end{array}
$$

$$
\left[\begin{array}{rrr|r}
1 & -3 & 2 & 8 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrl}
 & x & -3y & +2z=8. & E1 \\
 & & y & =0. & E7 \\
 & & & z=2. & E6 \\
-2(& & & z)=-2(2). & E6 \\
 & & -2z= & -4. & \\
 & x & -3y \quad 2z= & 8. & E1 \\
\hline
 & x & -3y \quad = & 4. & E8
\end{array}
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \leftarrow -2R_3 + R_1
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrrl}
 & x & -3y & +2z & = 8. & E1 \\
 & & y & & = 0. & E7 \\
 & & & z & = 2. & E6 \\
-2(& & & z &)=-2(2). & E6 \\
 & & & -2z & = -4. & \\
 & x & -3y & 2z & = 8. & E1 \\
\hline
 & x & -3y & & = 4. & E8 \\
 & 3(& & y &)=3(0). & E7 \\
 & & & 3y & = 0. & \\
 & & x & -3y & = 4. & E8 \\
\hline
 & & x & & = 4. & E9
\end{array}
$$

$$
\left[\begin{array}{ccc|c}
1 & -3 & 2 & 8 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 2
\end{array}\right]
\quad R_1 \leftarrow -2R_3 + R_1
$$

$$
\left[\begin{array}{ccc|c}
1 & -3 & 0 & 4 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 2
\end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrrl}
 & x & -3y & +2z & = 8. & E1 \\
 & & y & & = 0. & E7 \\
 & & & z & = 2. & E6 \\
-2(& & & z &) = -2(2). & E6 \\
 & & & -2z & = -4. & \\
 & x & -3y & 2z & = 8. & E1 \\
\hline
 & x & -3y & & = 4. & E8 \\
 & 3(& & y &) = 3(0). & E7 \\
 & & & 3y & = 0. & \\
 & & x & -3y & = 4. & E8 \\
\hline
 & & x & & = 4. & E9
\end{array}
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \leftarrow -2R_3 + R_1
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \leftarrow 3R_2 + R_1
$$

$$
\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]
$$

Matrix Notation

$$
\begin{array}{rrrrrl}
 & x & -3y & +2z & = 8. & E1 \\
 & & y & & = 0. & E7 \\
 & & & z & = 2. & E6 \\
-2(& & & z &)=-2(2). & E6 \\
 & & & -2z & = -4. & \\
\hline
 & x & -3y & 2z & = 8. & E1 \\
\hline
 & x & -3y & & = 4. & E8 \\
 & 3(& & y &)=3(0). & E7 \\
 & & & 3y & = 0. & \\
\hline
 & & x & -3y & = 4. & E8 \\
\hline
 & & x & & = 4. & E9
\end{array}
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \leftarrow -2R_3 + R_1
$$

$$
\left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \leftarrow 3R_2 + R_1
$$

$$
\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]
$$

Note that the above matrix notation means $x = 4$, $y = 0$, and $z = 2$. Thus the answer is

$$(4, 0, 2)$$