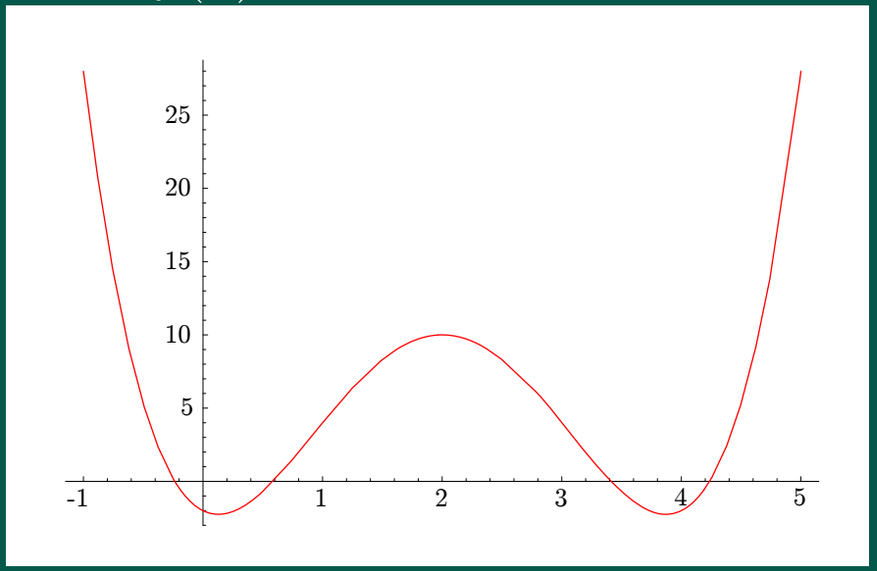
Finding Irrational Roots

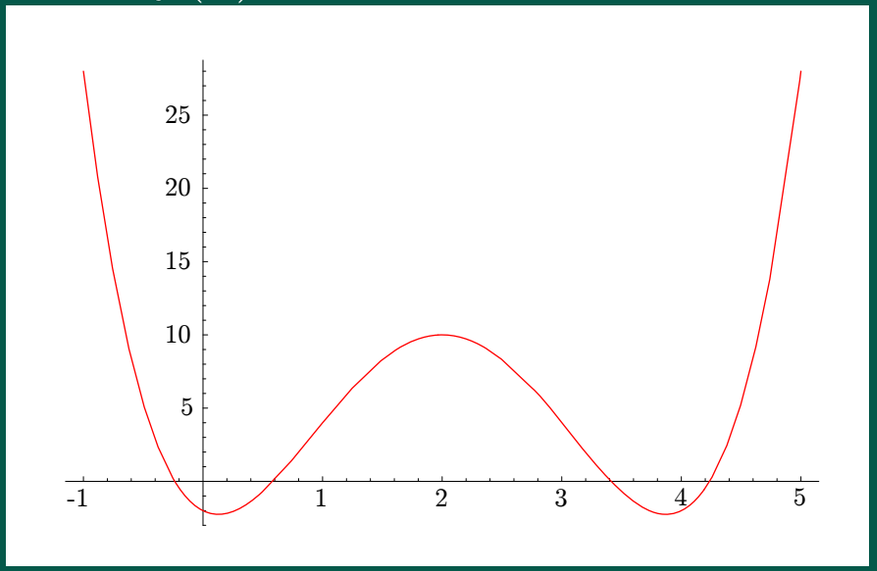
Goal: Find a root of the function $f(x) = x^4 - 8x^3 + 17x^2 - 4x - 2$.



Where does the function appear to be zero?

Finding Irrational Roots

Goal: Find a root of the function $f(x) = x^4 - 8x^3 + 17x^2 - 4x - 2$.

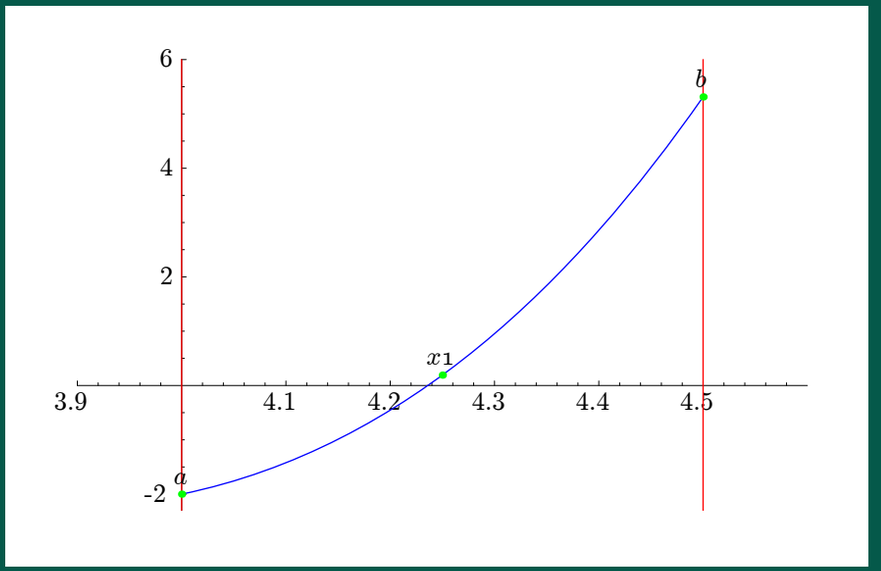


Where does the function appear to be zero? It appears to cross between -1/2 and 0, between 0 and 1, between 3 and 4, and between 4 and 4.5.

We will find the root between 4 and 4.5.

Finding Irrational Roots

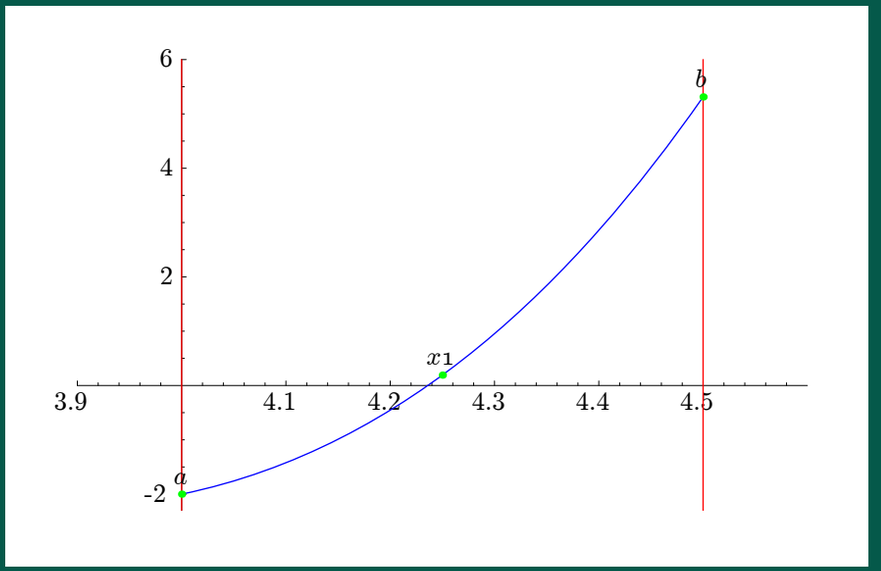
	x	$f(x)$
a	4.00000	-2.000000
b	4.50000	5.312500



Because f is negative at a and positive at b, it must be zero at some x value in between. What could we guess as the root?

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
b	4.50000	5.312500



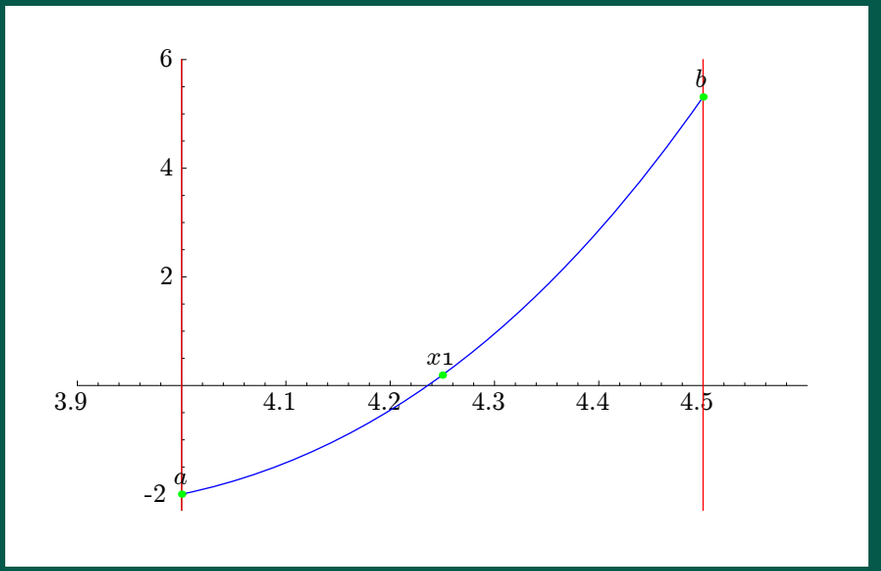
Because f is negative at a and positive at b, it must be zero at some x value in between. What could we guess as the root? The first estimate is

$$x_1 = \frac{a+b}{2} = 4.25.$$

What is the maximum possible error for this guess?

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
b	4.50000	5.312500



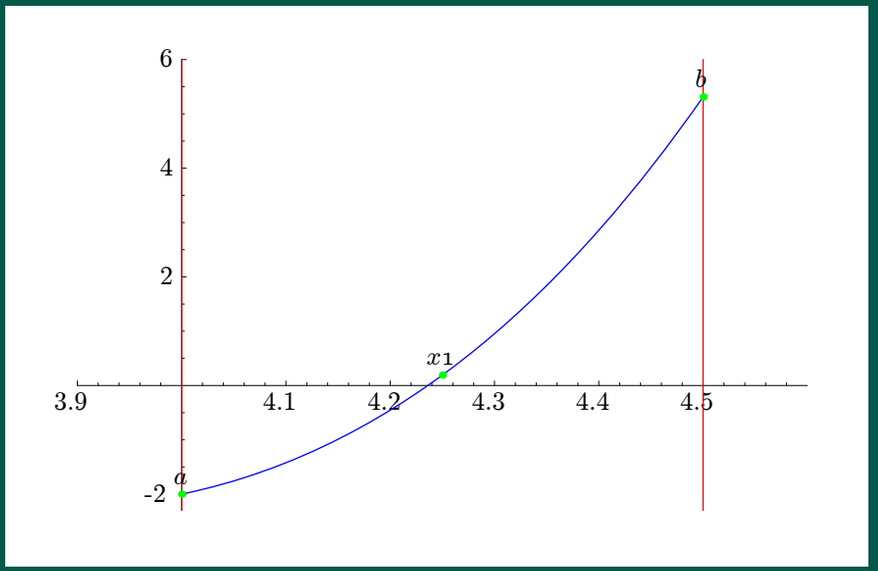
Because f is negative at a and positive at b, it must be zero at some x value in between. What could we guess as the root? The first estimate is

$$x_1 = \frac{a+b}{2} = 4.25.$$

What is the maximum possible error for this guess? If the root is near 4, then the error is approximately $|4.25 - 4| = 0.25$.

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
b	4.50000	5.312500



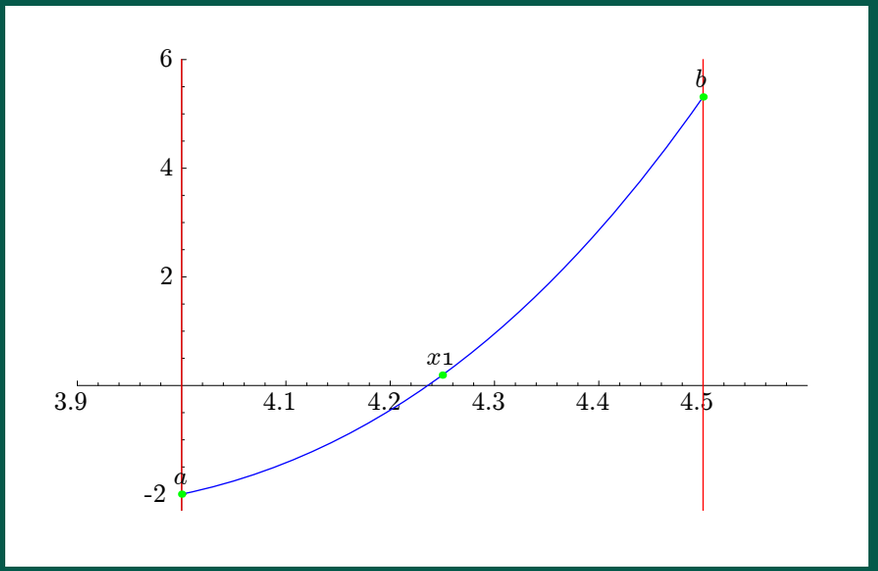
Because f is negative at a and positive at b, it must be zero at some x value in between. What could we guess as the root? The first estimate is

$$x_1 = \frac{a+b}{2} = 4.25.$$

What is the maximum possible error for this guess? If the root is near 4, then the error is approximately $|4.25 - 4| = 0.25$. If the root is near 4.5, then the error is approximately $|4.25 - 4.5| = 0.25$.

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
b	4.50000	5.312500



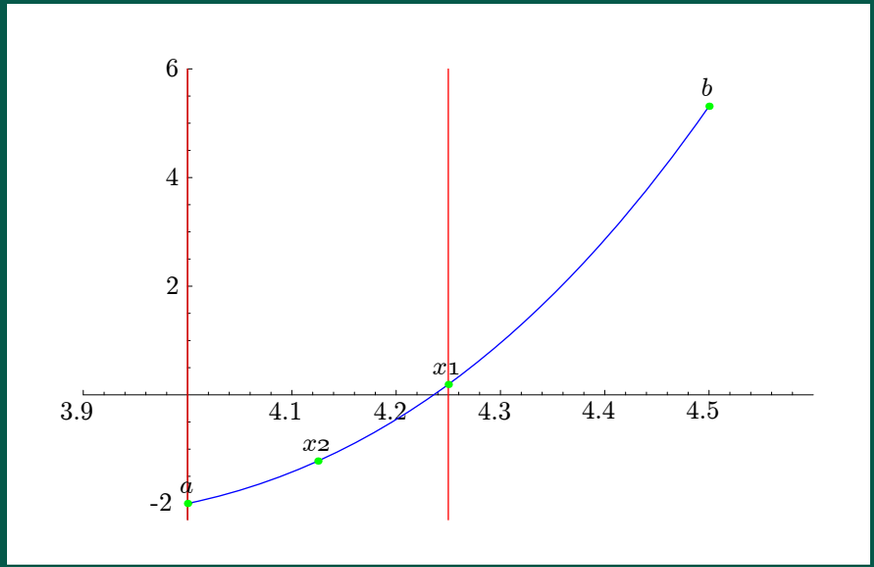
Because f is negative at a and positive at b, it must be zero at some x value in between. What could we guess as the root? The first estimate is

$$x_1 = \frac{a+b}{2} = 4.25.$$

What is the maximum possible error for this guess? If the root is near 4, then the error is approximately $|4.25 - 4| = 0.25$. If the root is near 4.5, then the error is approximately $|4.25 - 4.5| = 0.25$. Thus the maximum possible error is $1/4$.

Finding Irrational Roots

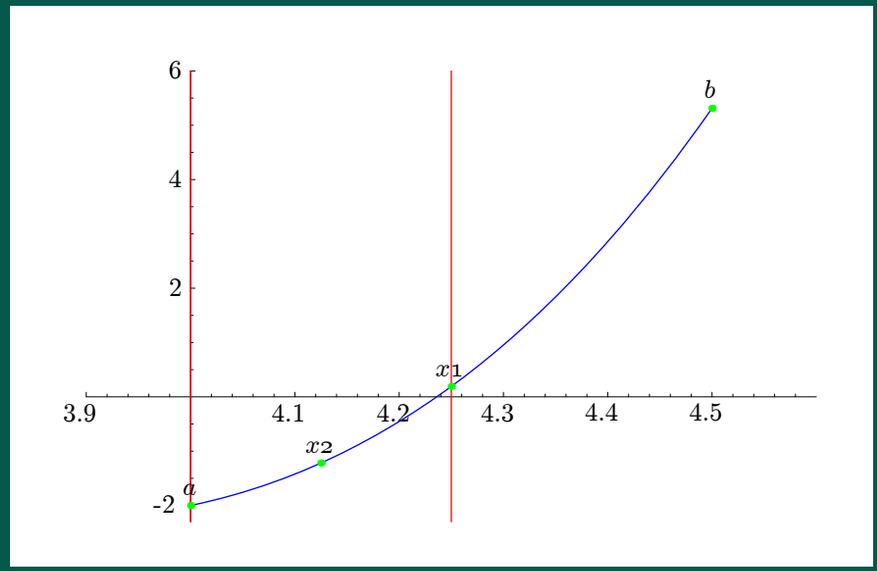
	x	$f(x)$
a	4.00000	-2.000000
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our second guess?

Finding Irrational Roots

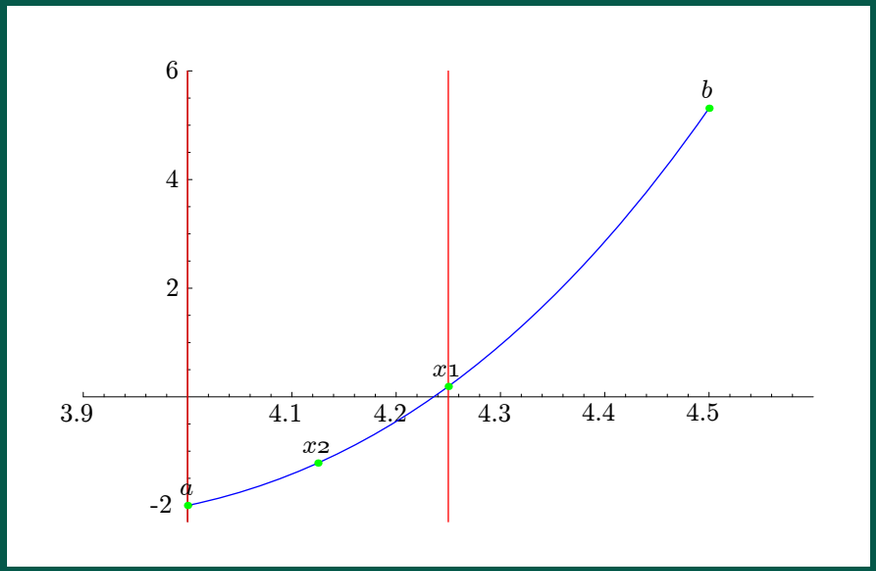
	x	$f(x)$
a	4.00000	-2.000000
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our second guess? Because f is negative at a and positive at x_1, it must be zero at some x value in between.

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_1	4.25000	0.191406
b	4.50000	5.312500



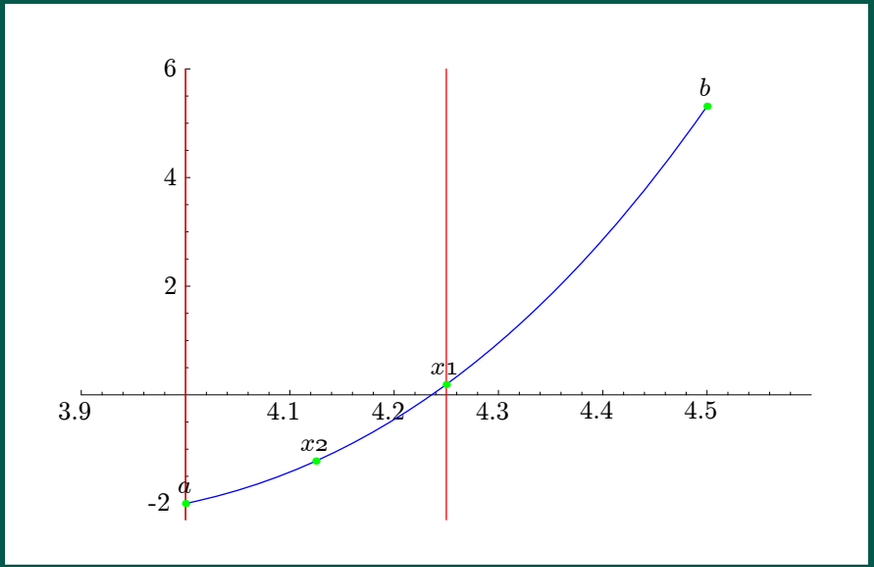
Where should we make our second guess? Because f is negative at a and positive at x_1, it must be zero at some x value in between. The second estimate is

$$x_2 = \frac{a + x_1}{2} = 4.125.$$

What is the maximum possible error for this guess?

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_1	4.25000	0.191406
b	4.50000	5.312500



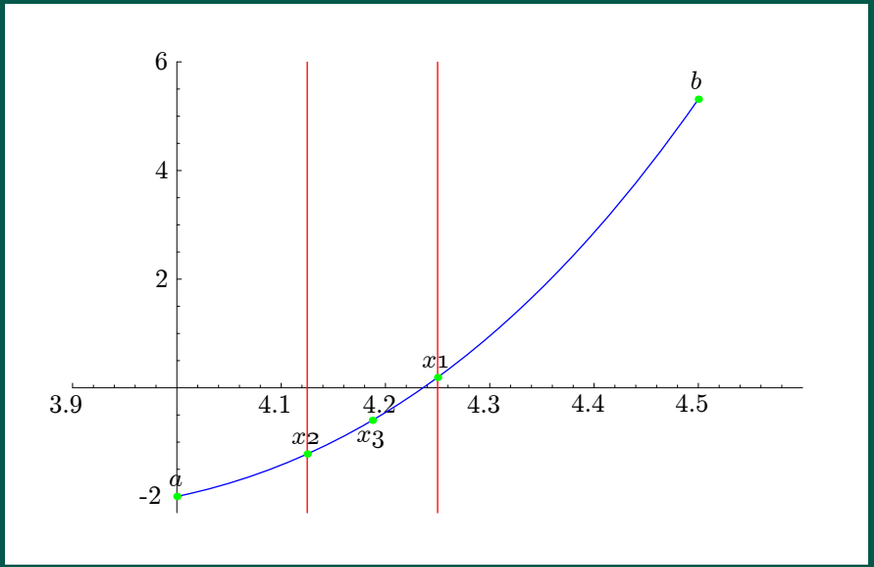
Where should we make our second guess? Because f is negative at a and positive at x_1, it must be zero at some x value in between. The second estimate is

$$x_2 = \frac{a + x_1}{2} = 4.125.$$

What is the maximum possible error for this guess? If the root is near 4, then the error is approximately $|4.125 - 4| = 0.125$. If the root is near 4.25, then the error is approximately $|4.125 - 4.25| = 0.125$. Thus the maximum possible error is $1/8$.

Finding Irrational Roots

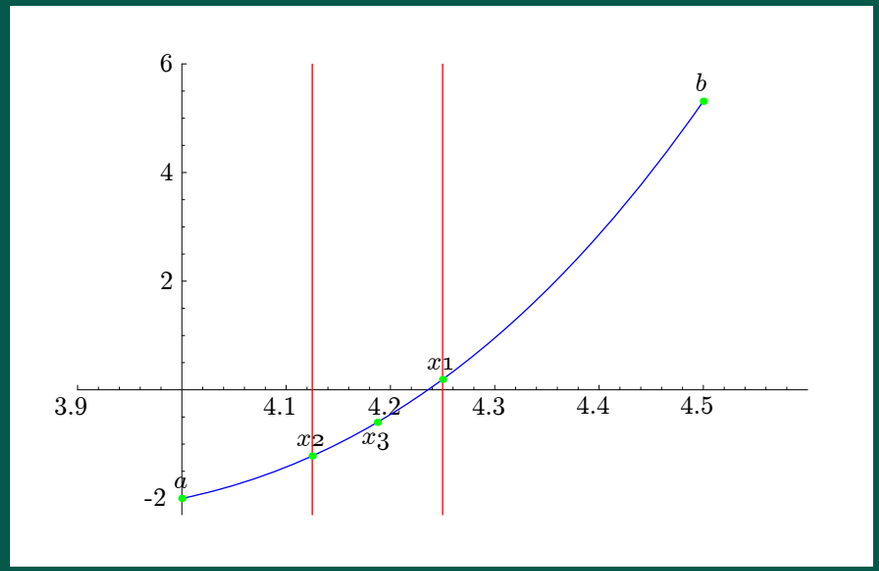
	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our third guess?

Finding Irrational Roots

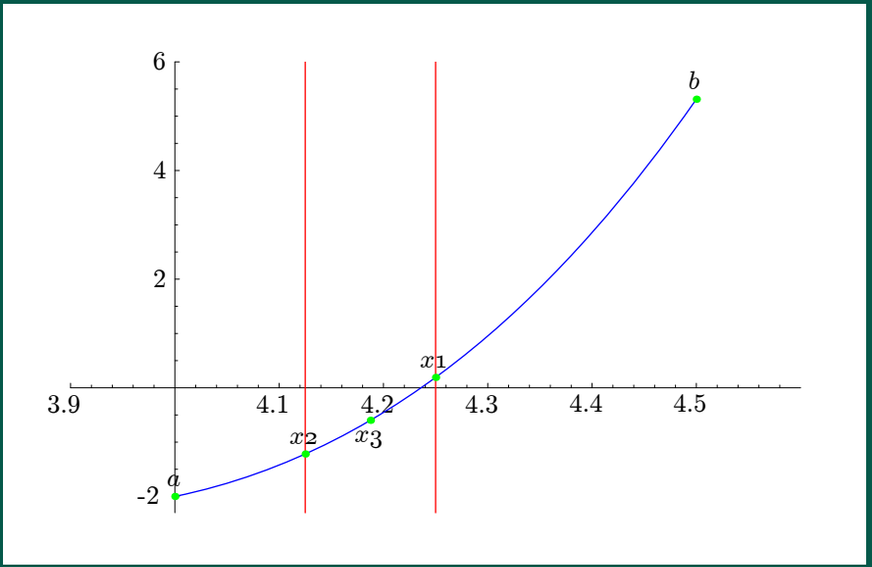
	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our third guess? Because f is negative at x_2 and positive at x_1, it must be zero at some x value in between.

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_1	4.25000	0.191406
b	4.50000	5.312500



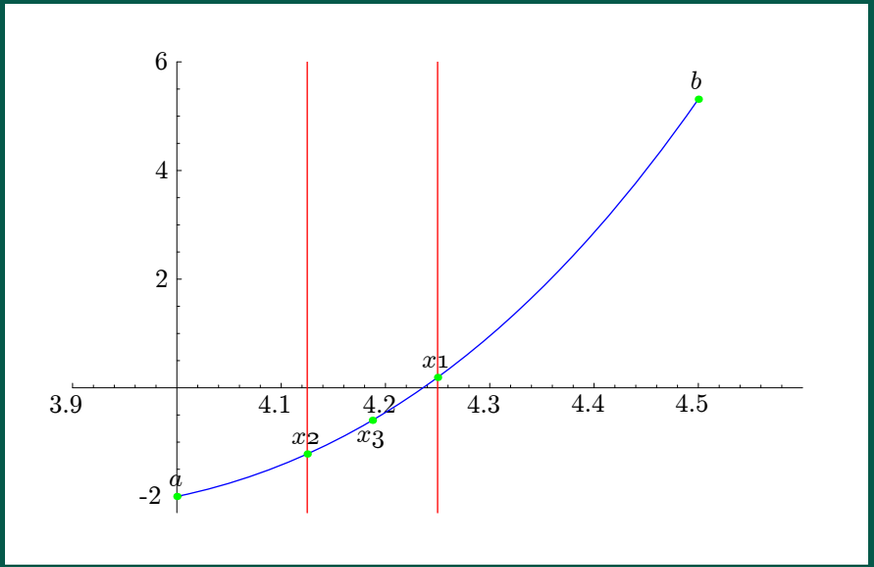
Where should we make our third guess? Because f is negative at x_2 and positive at x_1, it must be zero at some x value in between. The third estimate is

$$x_3 = \frac{x_2 + x_1}{2} = 4.1875.$$

What is the maximum possible error for this guess?

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_1	4.25000	0.191406
b	4.50000	5.312500



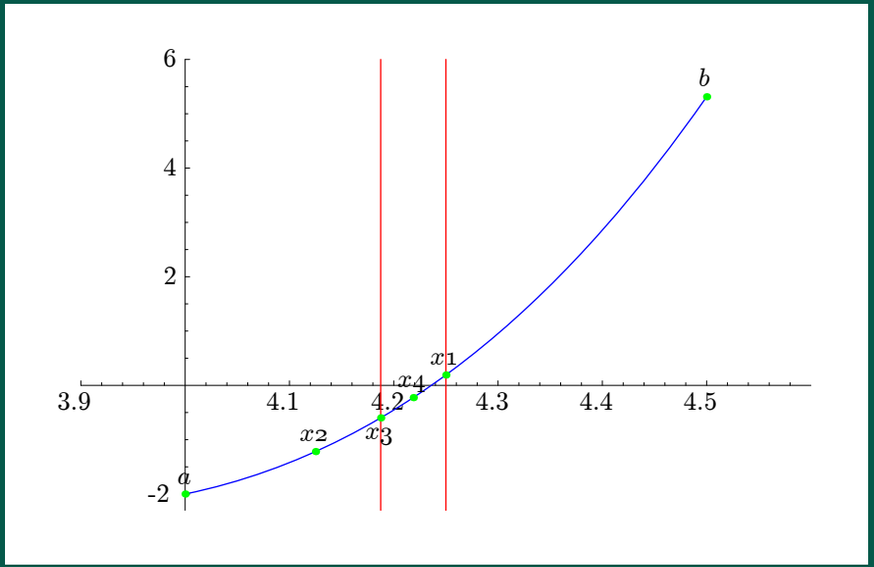
Where should we make our third guess? Because f is negative at x_2 and positive at x_1, it must be zero at some x value in between. The third estimate is

$$x_3 = \frac{x_2 + x_1}{2} = 4.1875.$$

What is the maximum possible error for this guess? If the root is near 4.25, then the error is approximately $|4.1875 - 4.25| = 0.0625$. If the root is near 4.125, then the error is approximately $|4.1875 - 4.125| = 0.0625$. Thus the maximum possible error is $1/16$.

Finding Irrational Roots

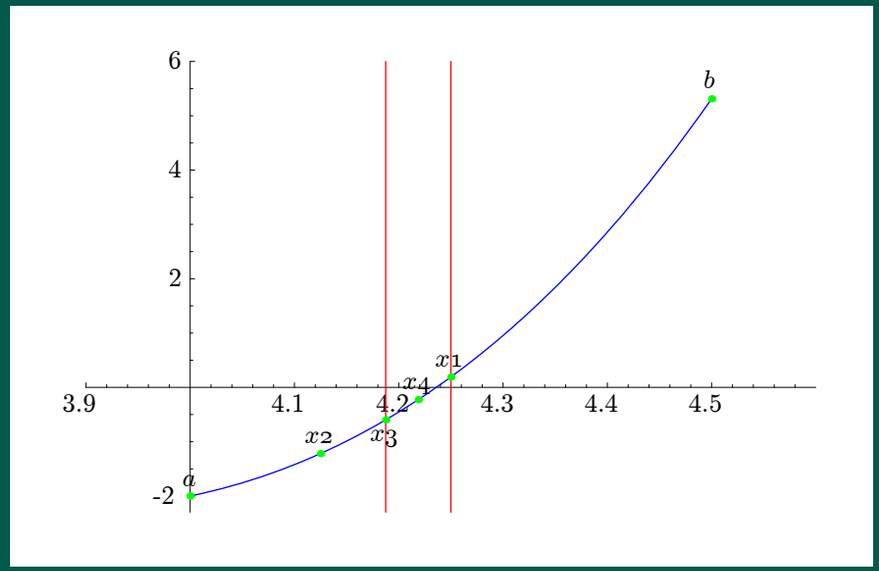
	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our fourth guess?

Finding Irrational Roots

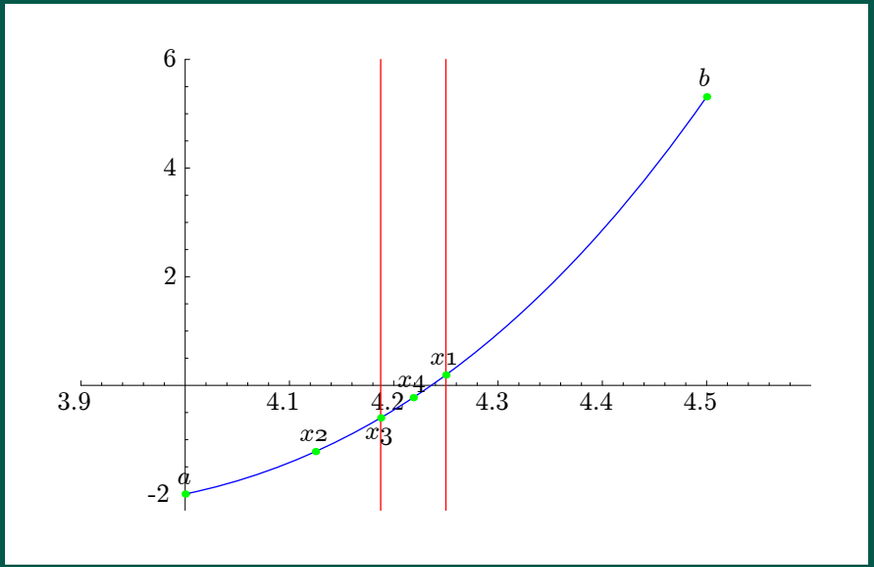
	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our fourth guess? Because f is negative at x_3 and positive at x_1, it must be zero at some x value in between.

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_1	4.25000	0.191406
b	4.50000	5.312500



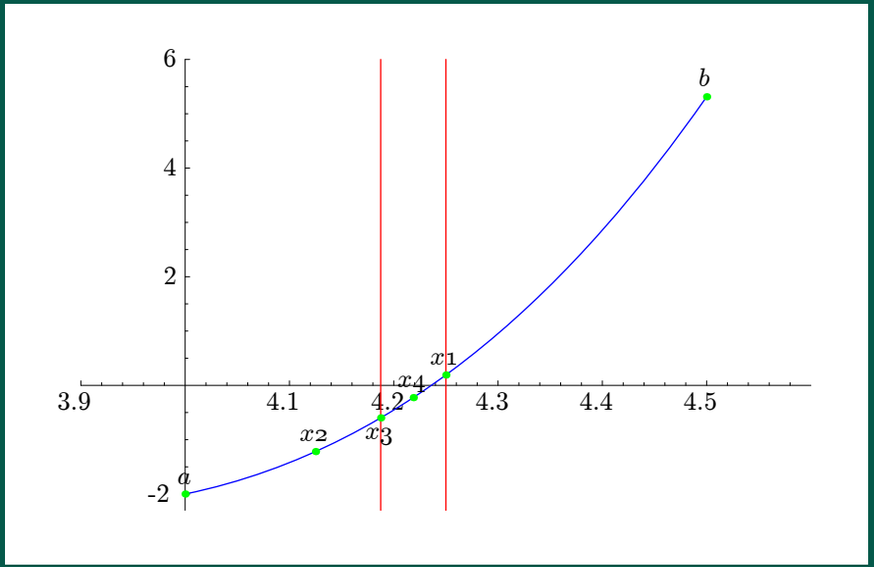
Where should we make our fourth guess? Because f is negative at x_3 and positive at x_1, it must be zero at some x value in between. The fourth estimate is

$$x_4 = \frac{x_3 + x_1}{2} = 4.21875.$$

What is the maximum possible error for this guess?

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_1	4.25000	0.191406
b	4.50000	5.312500



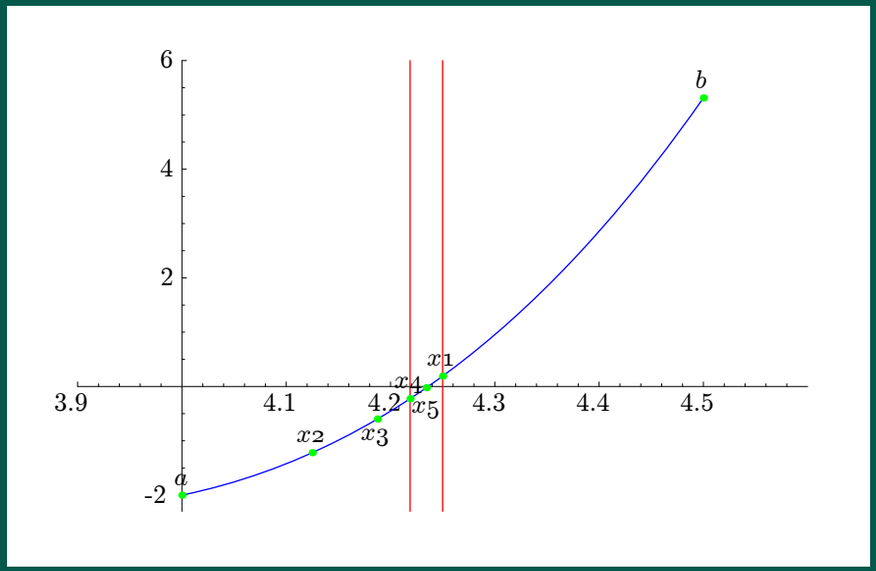
Where should we make our fourth guess? Because f is negative at x_3 and positive at x_1, it must be zero at some x value in between. The fourth estimate is

$$x_4 = \frac{x_3 + x_1}{2} = 4.21875.$$

What is the maximum possible error for this guess? If the root is near 4.25, then the error is approximately $|4.21875 - 4.25| = 0.03125$. If the root is near 4.1875, then the error is approximately $|4.21875 - 4.1875| = 0.03125$. Thus the maximum possible error is $1/32$.

Finding Irrational Roots

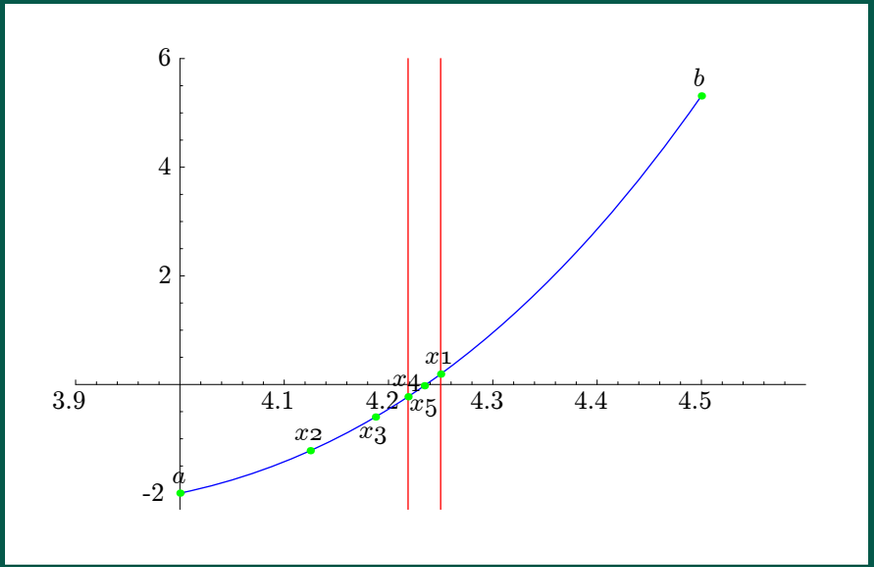
	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_4	4.21875	-0.225493
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our fifth guess?

Finding Irrational Roots

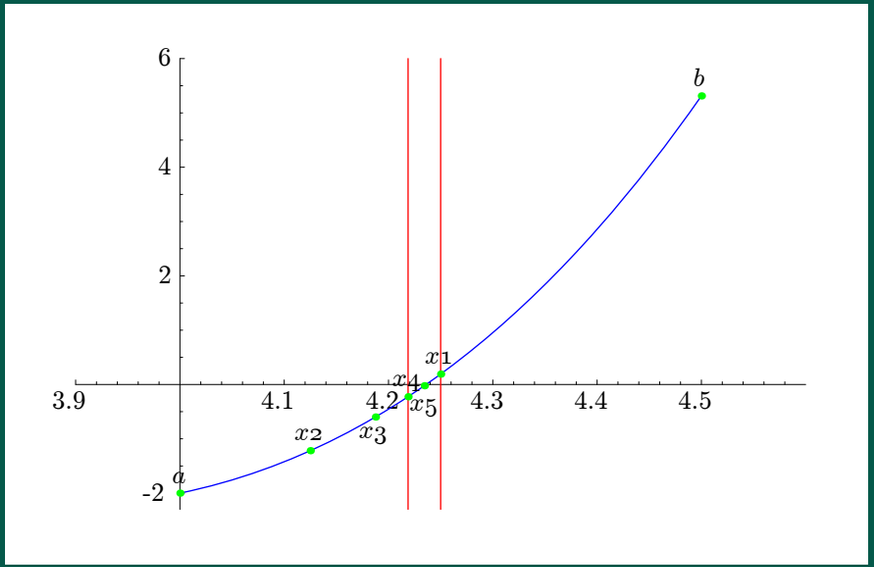
	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_4	4.21875	-0.225493
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our fifth guess? Because f is negative at x_4 and positive at x_1, it must be zero at some x value in between.

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_4	4.21875	-0.225493
x_1	4.25000	0.191406
b	4.50000	5.312500



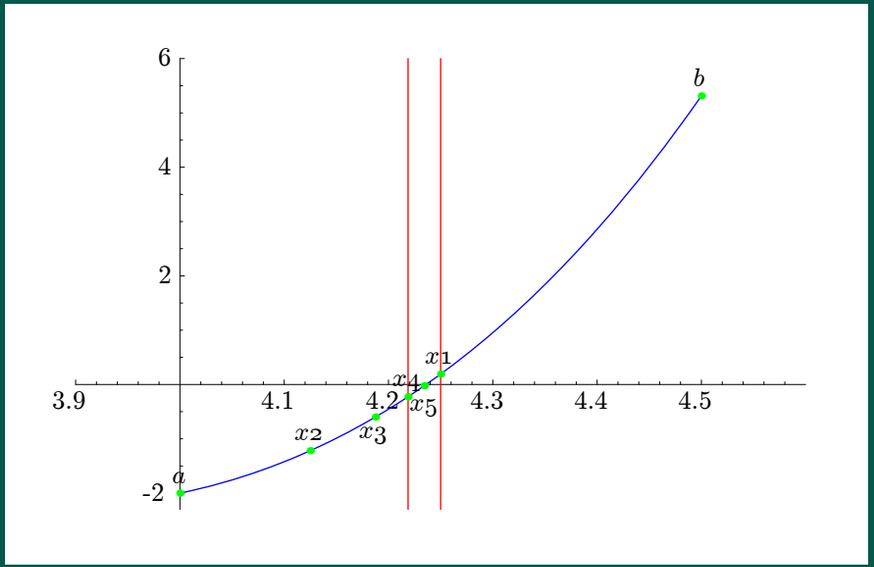
Where should we make our fifth guess? Because f is negative at x_4 and positive at x_1, it must be zero at some x value in between. The fifth estimate is

$$x_5 = \frac{x_4 + x_1}{2} = 4.23438.$$

What is the maximum possible error for this guess?

Finding Irrational Roots

	x	$f(x)$
a	4.00000	-2.000000
x_2	4.12500	-1.218510
x_3	4.18750	-0.598373
x_4	4.21875	-0.225493
x_1	4.25000	0.191406
b	4.50000	5.312500



Where should we make our fifth guess? Because f is negative at x_4 and positive at x_1, it must be zero at some x value in between. The fifth estimate is

$$x_5 = \frac{x_4 + x_1}{2} = 4.23438.$$

What is the maximum possible error for this guess? If the root is near 4.25, then the error is approximately $|4.23438 - 4.25| = 0.015625$. If the root is near 4.1875, then the error is approximately $|4.23438 - 4.21875| = 0.015625$. Thus the maximum possible error is $1/64$.