Goals

- Practice arithmetic (addition, subtraction, multiplication) with radicals by using old procedures
- Learn to divide some radical expressions

Addition

Addition with radicals is the same as previous addition (collecting like terms).

- Add $5x + 3x$.
- Add $5\sqrt{2} + 3\sqrt{2}$.
- Subtract $9x - 11x$.
- Subtract $9\sqrt{2} - 11\sqrt{2}$.

Reducing

To learn to reduce products and quotients involving radicals note the comparisons below. These indicate where order (of operations) does not matter.

- What is $\sqrt{4}\sqrt{4}$? Note order of operations requires computing each square root first.
- What is $\sqrt{4 \cdot 4}$? Note order of operations requires multiplying first.
- Based on this what is $\sqrt{5}\sqrt{5}$? Does the usual order of operations matter?
- What is $\frac{\sqrt{16}}{\sqrt{4}}$? Note order of operations requires computing each square root first.
- What is $\sqrt{\frac{16}{4}}$? Note order of operations requires computing the quotient first.
- Based on this what is $\frac{\sqrt{45}}{\sqrt{5}}$? Does the usual order of operations matter?

Reducing

The steps below introduce how to reduce, as much as possible, a root of a larger expression.

- Factor 36.
- What is $\sqrt{36}$?
- Factor 1600.
- What is $\sqrt{1600}$?
- Based on these two examples, how might you calculate radicals even for numbers that don't have a nice square root?
- Factor 1100.
- What is $\sqrt{1100}$?
- What is $\sqrt{x^{26}}$?
- Simplify $\sqrt{x^{27}}$.

Multiplication

Multiplication with radicals is the same as previous multiplication (distribution).

- Multiply $(x+8)(x-3)$
- Multiply $(\sqrt{5}+8)(\sqrt{5}-3)$

Division

The following procedure is often useful including to divide expressions with radicals.

$$\frac{2+\sqrt{3}}{5-\sqrt{3}} = \text{multiply using the 'conjugate'}$$

$$\frac{2+\sqrt{3}}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}} =$$

$$\frac{10+2\sqrt{3}+5\sqrt{3}+\sqrt{3}^2}{25+5\sqrt{3}-5\sqrt{3}-\sqrt{3}^2} =$$

$$\frac{10+7\sqrt{3}+3}{25-3} =$$

$$\frac{13+7\sqrt{3}}{22} =$$

$$\frac{13}{22}+\frac{7}{22}\sqrt{3}.$$