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Testing a Potential Basis

$$\mathcal{B} = \{6, -3 + 6x, 1 - 6x + 6x^2\}$$

Test if $\mathcal{B}$ is a basis for P^2 (polynomials of degree 2 or less).

Independence

The test for independence begins, as always, with the definition of independence.

$$\begin{aligned}
a(6) + b(-3 + 6x) + c(1 - 6x + 6x^2) &= 0. \\
6a - 3b + c &= 0. \\
6b - 6c &= 0. \\
6c &= 0.
\end{aligned}$$

$$\begin{bmatrix} 6 & -3 & 1 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & 0 & 6 & 0 \end{bmatrix} \sim \quad R_1 \tfrac{1}{6} \leftarrow R_1$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 6 & -6 & 0 \\ 0 & 0 & 6 & 0 \end{bmatrix} \sim \quad \begin{array}{l} R_1 \leftarrow \frac{1}{12} R_2 + R_1 \\ R_2 \frac{1}{6} \leftarrow R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 6 & 0 \end{bmatrix} \sim \quad \begin{array}{l} R_1 \leftarrow \frac{1}{18} R_3 + R_1 \\ R_2 \leftarrow \frac{1}{6} R_3 + R_2 \\ R_3 \frac{1}{6} \leftarrow R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Thus the only solution is the trivial solution.

Span

The test for span begins, as always, with the definition of spanning

$$\begin{aligned}
a(6) + b(-3 + 6x) + c(1 - 6x + 6x^2) &= r_0 + r_1 x + r_2 x^2. \\
6a - 3b + c &= r_0. \\
6b - 6c &= r_1. \\
6c &= r_2.
\end{aligned}$$

Note that this is the same matrix that was row reduced to check independence. Additionally the number of solutions to a non-homogeneous (this system) equals the number of solutions for the homogeneous (previous system). Thus a solution exists and is unique. This is a spanning set.